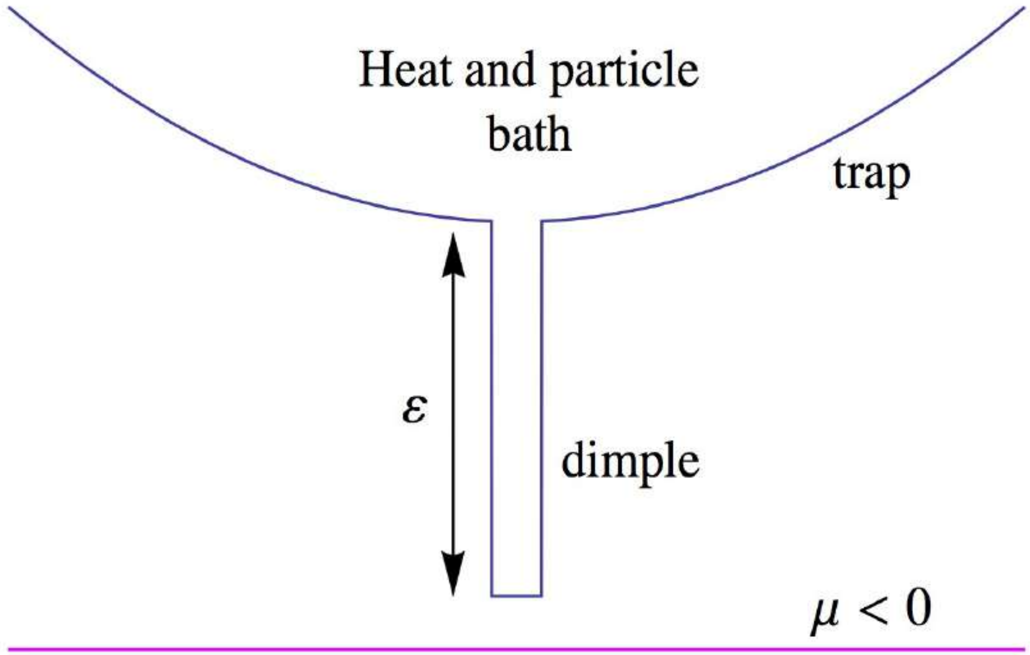
# Kinetics of Bose condensation in a dimple potential

Shovan Dutta

# Experimental system




Typical initial condition:

$$\mathcal{N} \approx 5 \times 10^8$$

$$T \approx 1\ \mu\text{K}$$

$$\rho_{\text{bath}} \lambda_T^3 \approx 0.04 \ll 1$$

$$\Longrightarrow \quad \text{bath is classical}$$

$$\langle n_E \rangle = \frac{1}{e^{\beta(E-\mu)} - 1}$$

$$\approx e^{-\beta(E-\mu)}$$

Goal: Induce fast condensation in a narrow dimple potential by increasing particle density

# Model

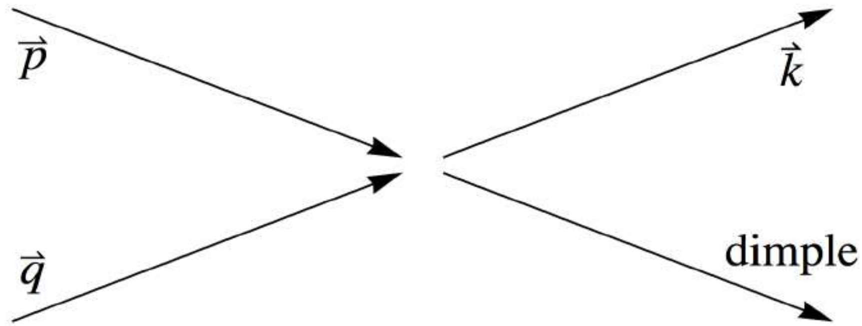
- The bath: classical Boltzmann gas in a harmonic well having a thermal distribution
- The dimple: quantum Bose gas in a square well
- The bath sets T and $\mu$
- Rough idea: assuming thermal equilibrium between bath and dimple,

$$\langle n_E \rangle = \frac{1}{e^{\beta(E-\mu)} - 1}$$

$\Longrightarrow$ for condensation, $\mu \approx -\varepsilon_{\mathrm{dimple}}$

# Infinite trap: kinetics

- Two-body collision




- Rate proportional to

  i) occupation: $e^{-\beta\left(\frac{p^2+q^2}{2m}-2\mu\right)}$

  ii) Bose stimulation: $1+N_{\vec{n}}$

  iii) Fermi's golden rule: $\frac{2\pi}{\hbar}\left(\frac{4\pi\hbar^2 a_s}{m}\right)^2 |\tilde{\psi}_{\vec{n}}(\vec{p}+\vec{q}-\vec{k})|^2$

# Infinite trap: kinetics

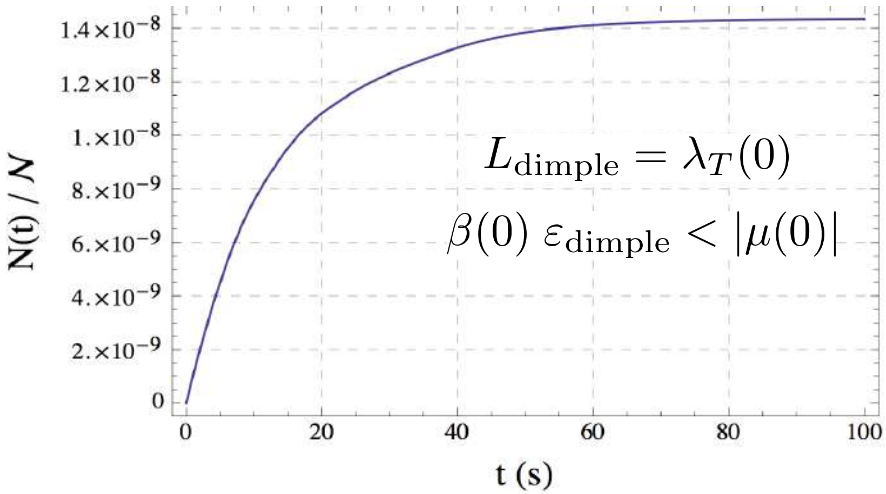
$$\psi_{\vec{n}}(\vec{r}) = \frac{1}{L_{\text{dimple}}^{3/2}} e^{i\frac{2\pi}{L_{\text{dimple}}}\vec{n}.\vec{r}} \quad ; \quad \varepsilon_{\vec{n}} = -\varepsilon_{\text{dimple}} + \frac{2\pi^2\hbar^2}{mL_{\text{dimple}}^2}\vec{n}^2$$

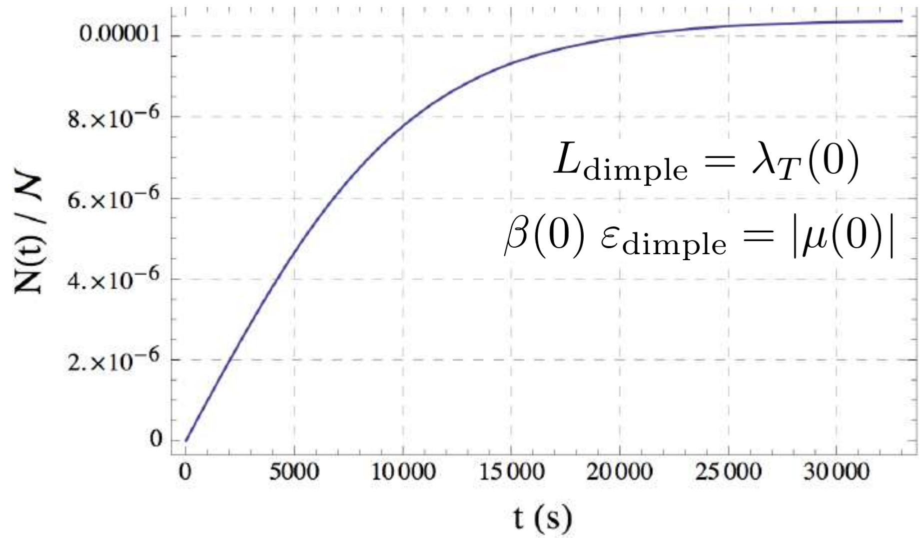
$$\left(\frac{dN_{\vec{n}}}{dt}\right)_{in} = \frac{2\pi}{\hbar}\left(\frac{4\pi\hbar^2 a_s}{m}\right)^2 \int \frac{d^3p d^3q}{(2\pi\hbar)^6}\, e^{-\beta(\frac{p^2+q^2}{2m}-2\mu)}\, (1+N_{\vec{n}})$$
$$\times\, \delta\Big((p^2+q^2-(\vec{p}+\vec{q}-2\pi\vec{n}/L_{\text{dimple}})^2)/2m+\varepsilon_{\vec{n}}\Big)$$

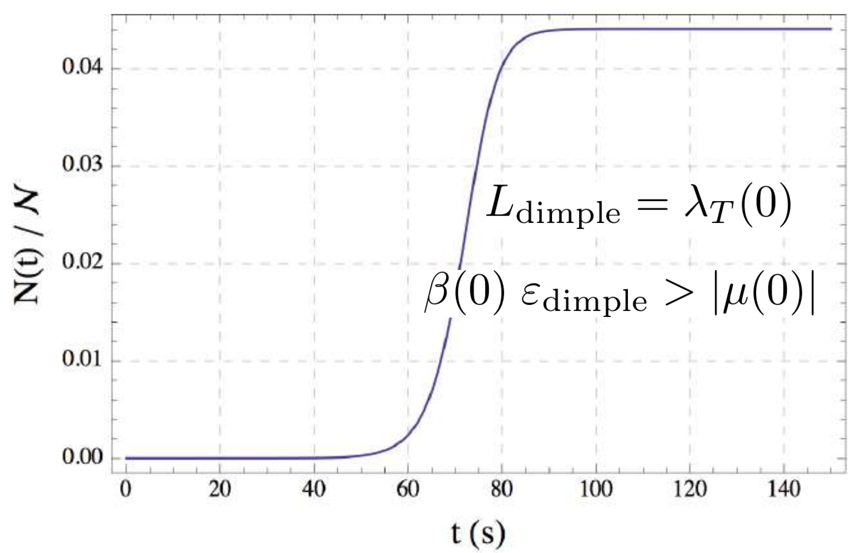
$$\left(\frac{dN_{\vec{n}}}{dt}\right)_{out} = -\frac{2\pi}{\hbar}\left(\frac{4\pi\hbar^2 a_s}{m}\right)^2 \int \frac{d^3p d^3q}{(2\pi\hbar)^6}\, e^{-\beta(\frac{k^2}{2m}-\mu)}\, N_{\vec{n}}$$
$$\times\, \delta\Big((p^2+q^2-(\vec{p}+\vec{q}-2\pi\vec{n}/L_{\text{dimple}})^2)/2m+\varepsilon_{\vec{n}}\Big)$$

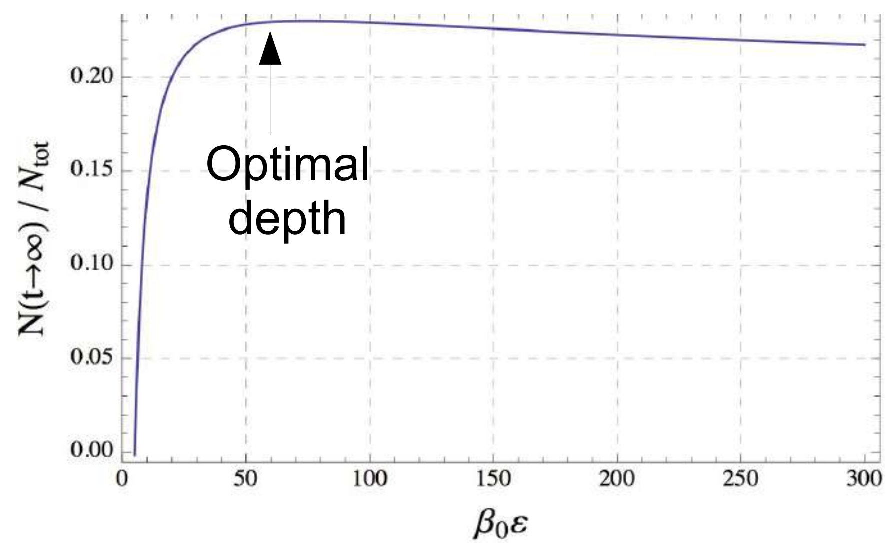
$$N_{\text{bath}}(t) = \mathcal{N} - N_{\text{dimple}}(t) \quad ; \quad \dot{E}_{\text{bath}}(t) = -\varepsilon_{\vec{n}}(t)\, \dot{N}_{\text{dimple}}(t)$$

# Time evolution for one bound state

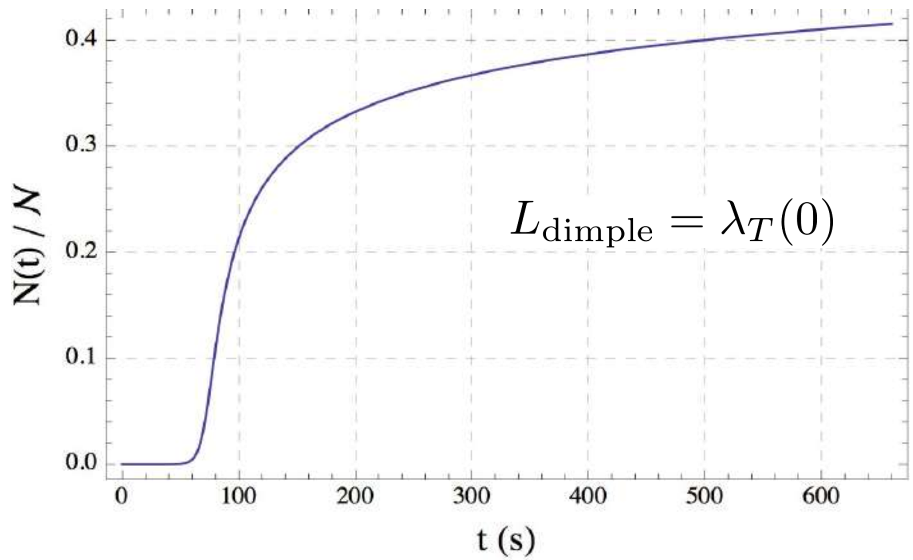


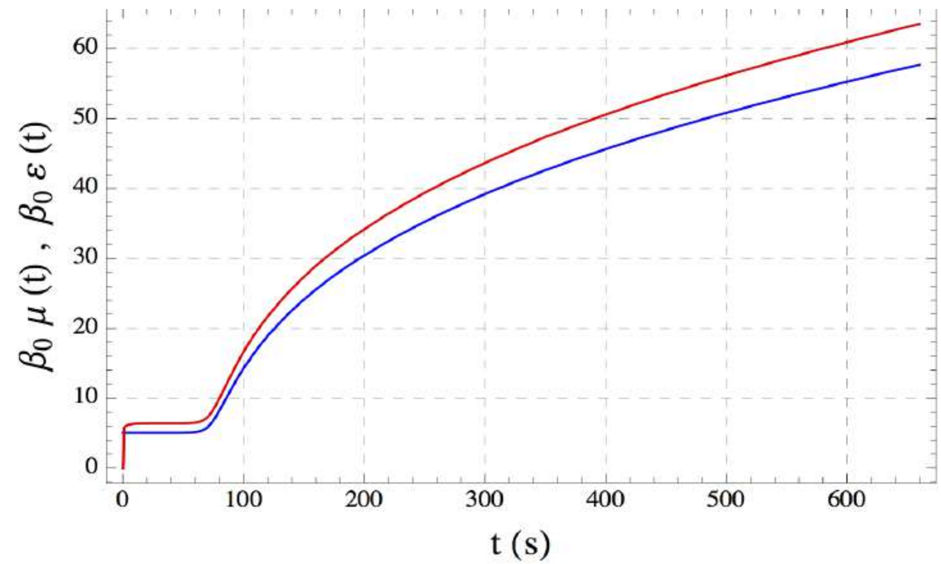



# Optimal evolution for one bound state



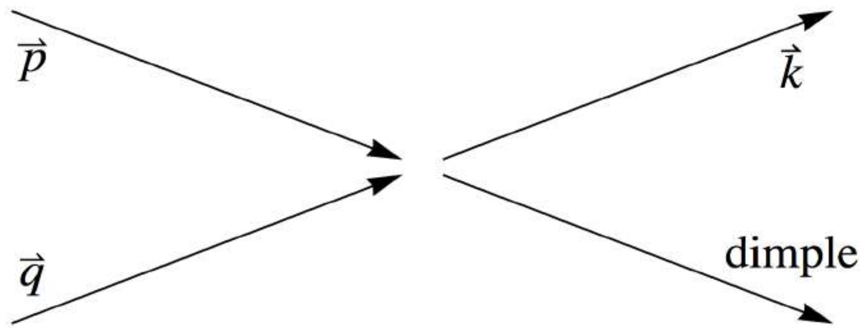




- However, in typical experiments, $L_{\text{dimple}} \gg \lambda_T(0)$ and $\beta(0)\,\varepsilon_{\text{dimple}} \gtrsim 5$

$\Longrightarrow$ number of bound states $\approx \sqrt{\dfrac{\beta\varepsilon_{\text{dimple}}}{\pi}}\dfrac{L_{\text{dimple}}}{\lambda_T} \gg 1$

# Many bound states: interaction kinetics

- Transition between two energy levels:




- **Rate** $\propto \exp\left[-\beta(p^2/2m-\mu)\right] N_{\vec{n}}$ **and** $1+N_{\vec{m}}$.

$$\frac{dN_{\vec{n}\to\vec{m}}}{dt} = \frac{2\pi}{\hbar}\left(\frac{4\pi\hbar^2 a_s}{m}\right)^2\left(\frac{2\pi\hbar}{L_{\mathrm{dimple}}}\right)^3 \int \frac{d^3p d^3k}{(2\pi\hbar)^6}\, e^{-\beta(\frac{p^2}{2m}-\mu)}$$
$$\times\; N_{\vec{n}}\,(1+N_{\vec{m}})\,\delta(p^2/2m+\varepsilon_{\vec{n}}-k^2/2m-\varepsilon_{\vec{m}})$$
$$\times\; \delta^3(\vec{p}+2\pi\vec{n}/L_{\mathrm{dimple}}-\vec{k}-2\pi\vec{m}/L_{\mathrm{dimple}})$$

# Many bound states: continuum approximation

$$\sum_{\vec{n}} N_{\vec{n}}(t) \longrightarrow \mathcal{N} \int f\big(\beta(0)E, t\big)\ d(\beta(0)E)$$

$$f\big(\beta(0)E, t\big) = g\big(\beta(0)E_{\vec{n}}\big)\ N_{\vec{n}}(t)/\mathcal{N}$$

where $g\big(\beta(0)E\big) = 2\ (L_{\text{dimple}}/\lambda_T(0))^3 \sqrt{\beta(0)E/\pi}$ is the density of states from the bottom of the dimple.

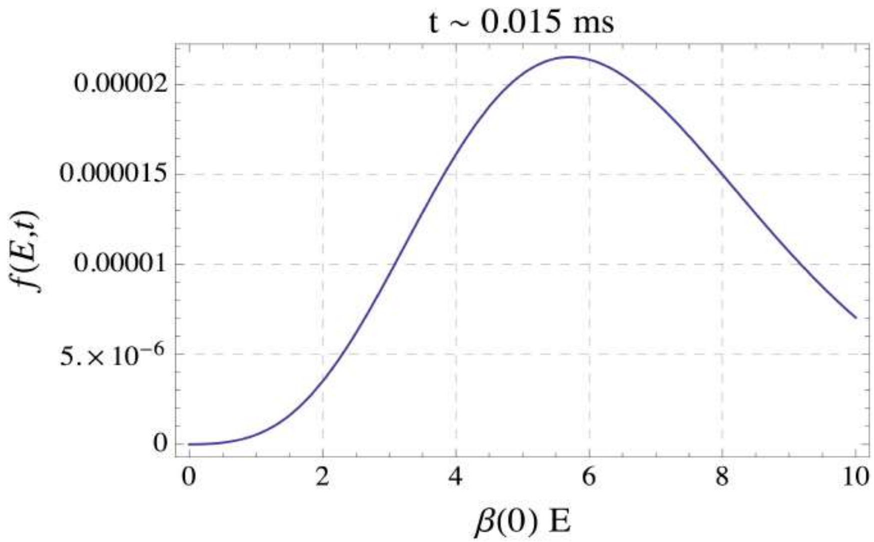
- Consider the zero energy state separately to observe condensation.
- Condensation occurs when $\rho_{\text{bath}}(0)\lambda_T^3(0)\ (V_{\text{bath}}(0)/V_{\text{dimple}}) \gg 1$ as expected.

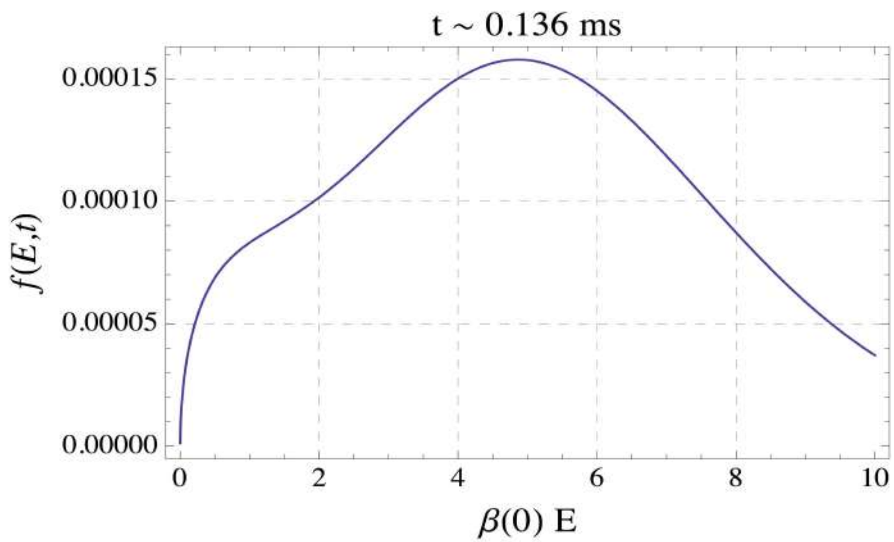
# Interaction causes thermal equilibrium within dimple

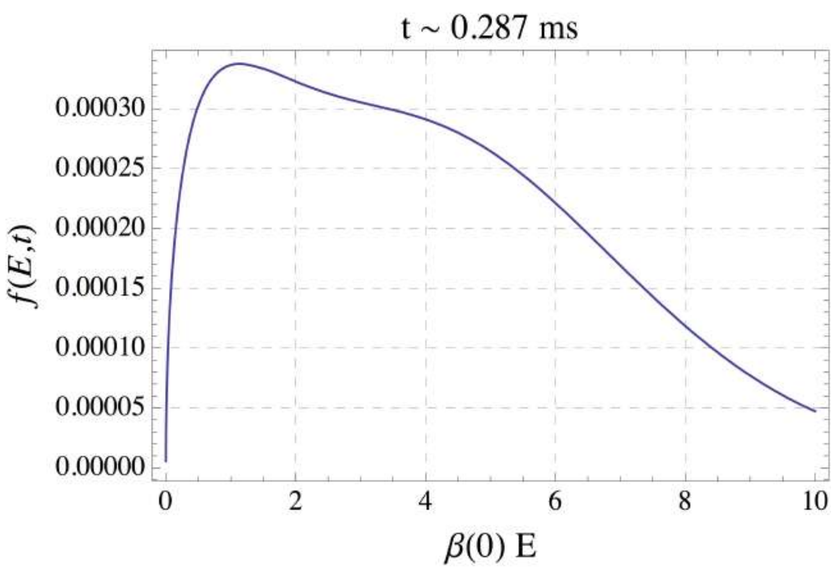
$$\beta(0)\ \varepsilon_{\mathrm{dimple}} = 10$$

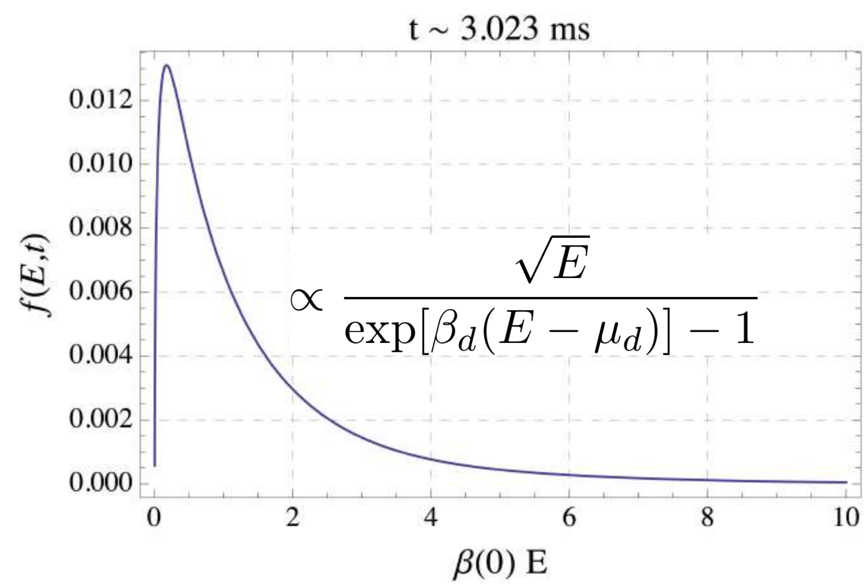
$$\rho_{\mathrm{bath}}(0)\lambda_T^3(0) \approx 0.05$$

$$V_{\mathrm{bath}}(0)/V_{\mathrm{dimple}} = 2000$$



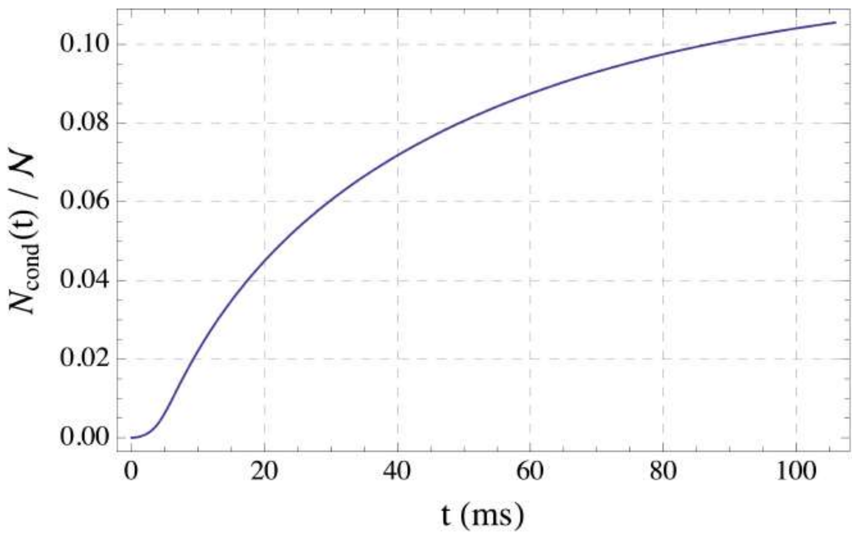



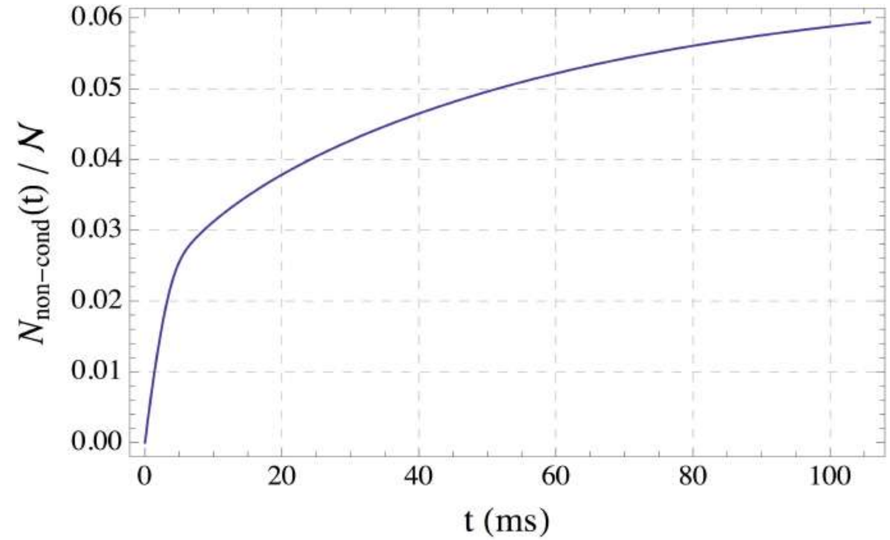
# Time evolution of physical quantities

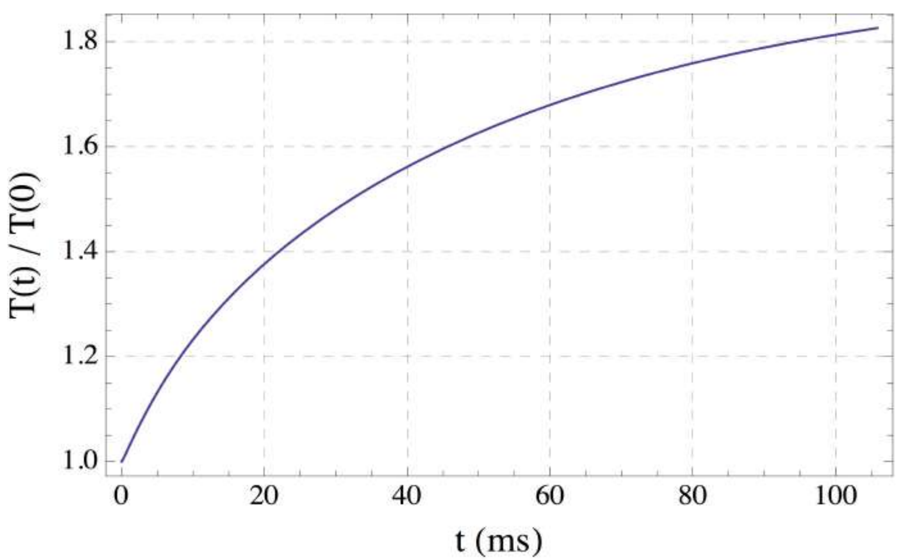
$$\beta(0)\,\varepsilon_{\rm dimple} = 10$$

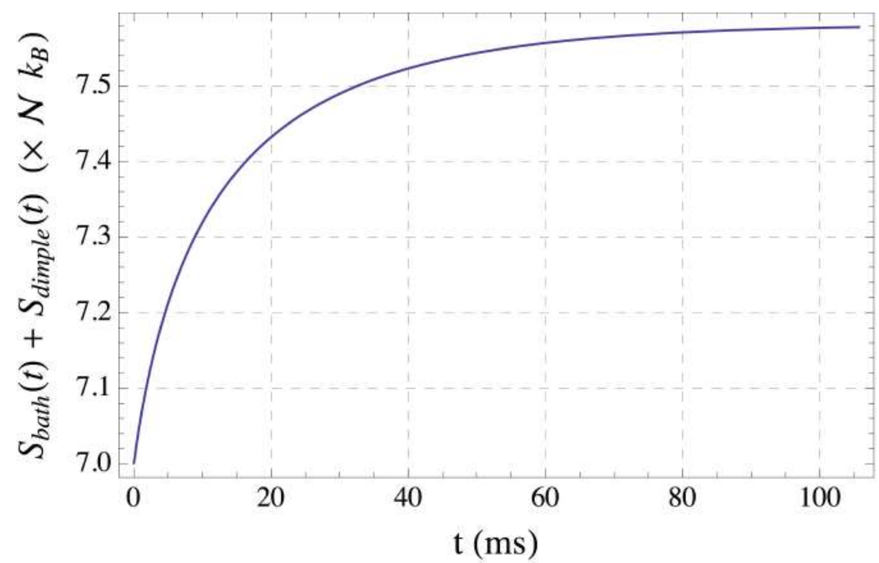
$$\rho_{\rm bath}(0)\lambda_T^3(0) \approx 0.05$$

$$V_{\rm bath}(0)/V_{\rm dimple} = 2000$$



# Thermal approximation for dimple

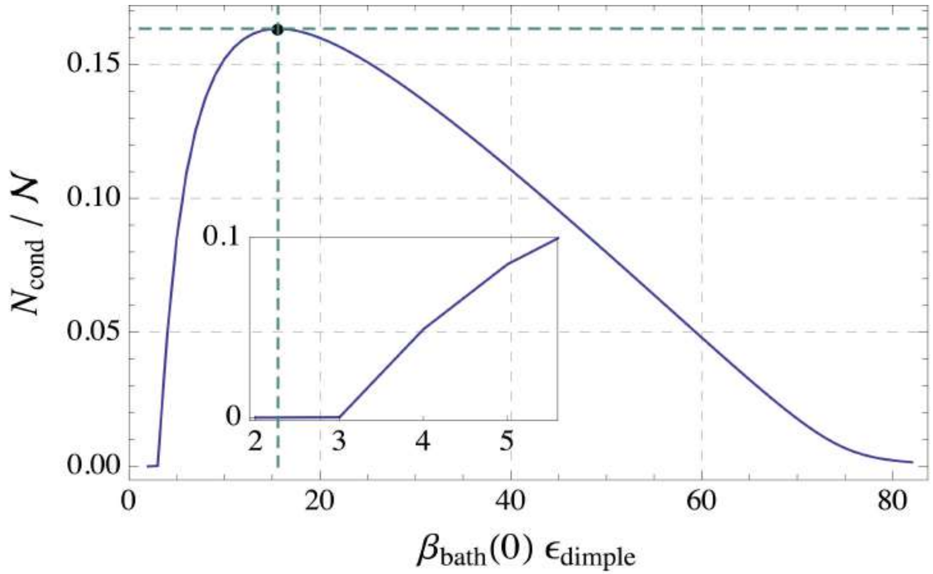
- The dimple very quickly reaches thermal equilibrium due to interactions.
- $\Longrightarrow$ Simplified model: assume a thermal distribution

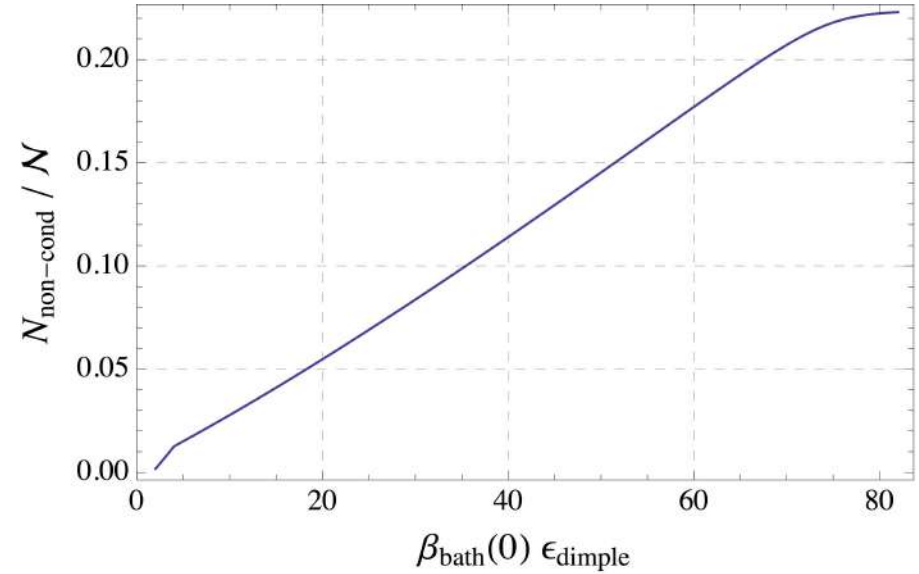
$$f\left(\beta(0)E, t\right) = \frac{V_{\text{dimple}}}{\mathcal{N}\lambda_T^3(0)} \frac{2\sqrt{\beta(0)E/\pi}}{\exp[\beta_d(t)(E-\mu_d(t))]-1}$$

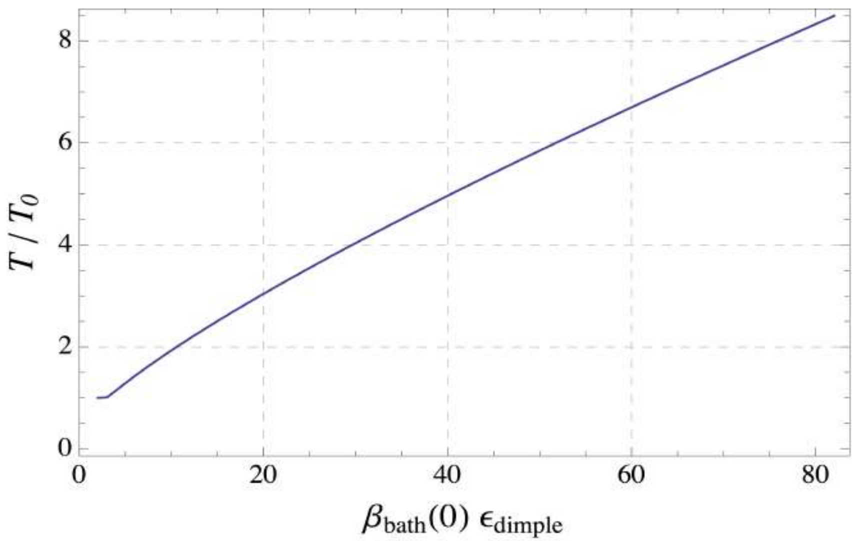
  and see how $\beta_d(t)$ and $\mu_d(t)$ vary with time.
- This method reproduces all features of the general model following thermal equilibrium.

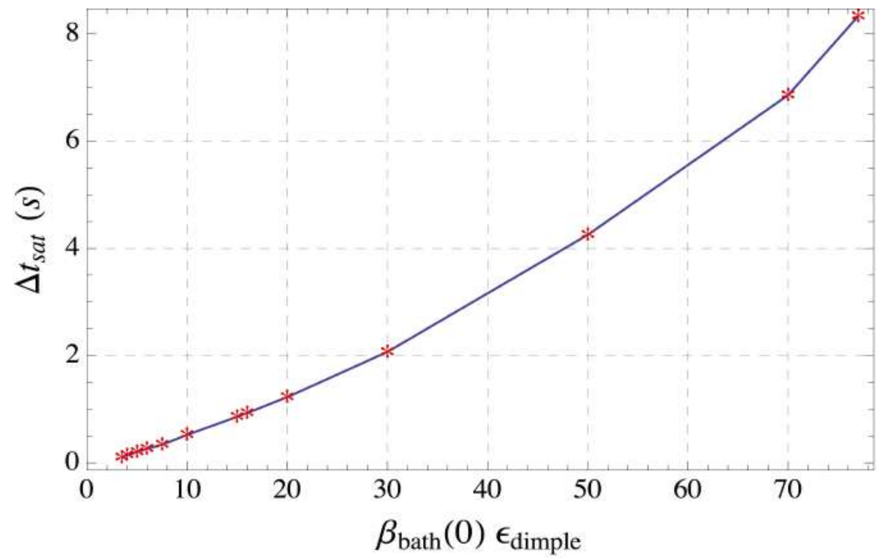
# Variation with dimple depth

$$\rho_{\text{bath}}(0)\lambda_T^3(0) = e^{-3}$$
$$V_{\text{bath}}(0)/V_{\text{dimple}} = 5000$$



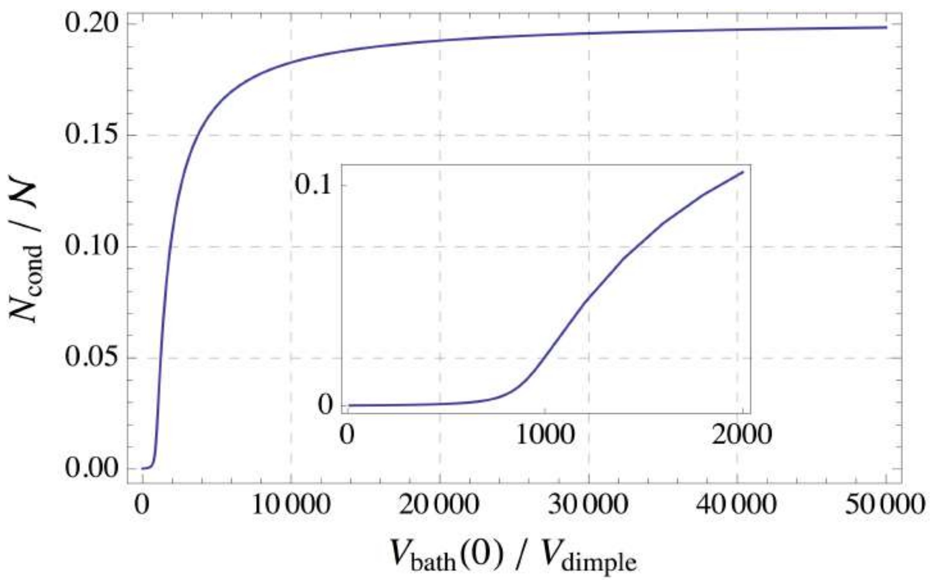



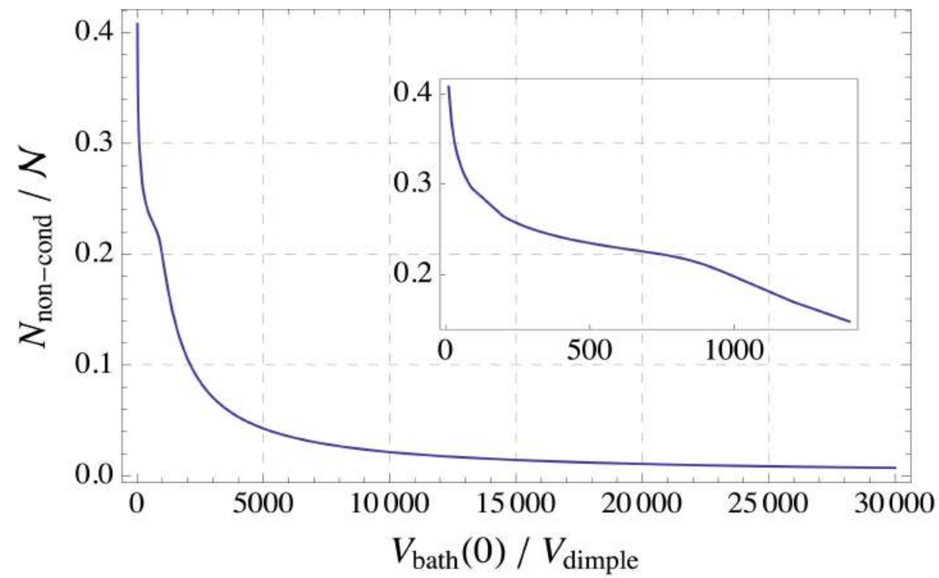



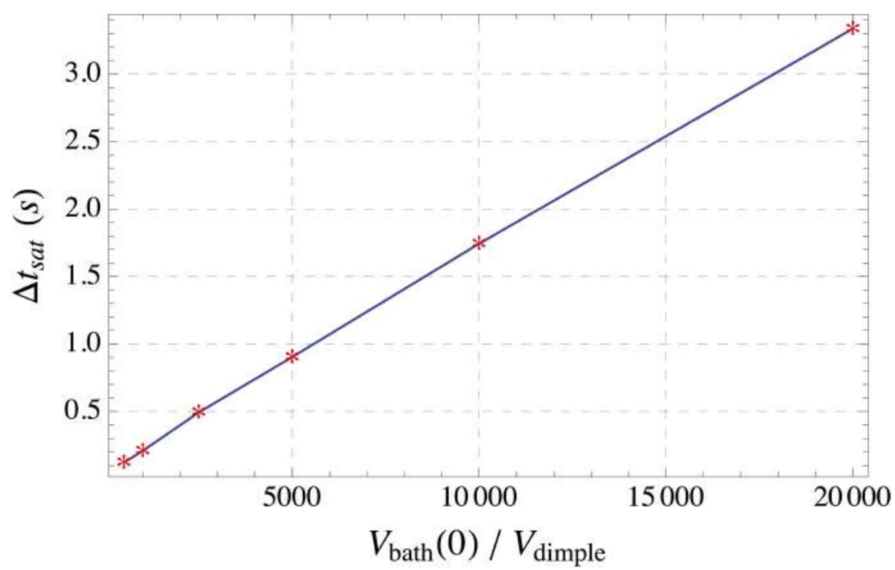



# Variation with volume ratio

$$\rho_{\text{bath}}(0)\lambda_T^3(0) = e^{-3}$$
$$\beta(0)\ \varepsilon_{\text{dimple}} \approx 15.6$$



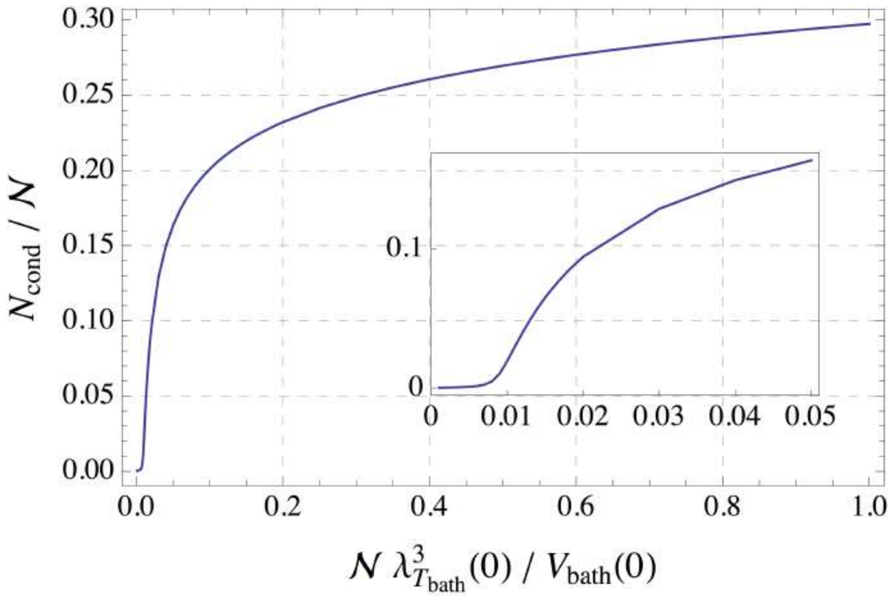



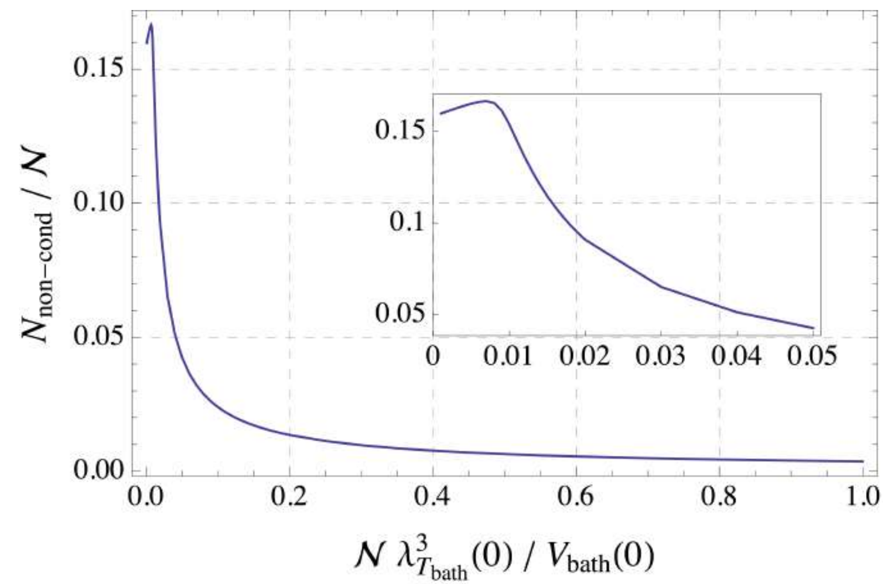



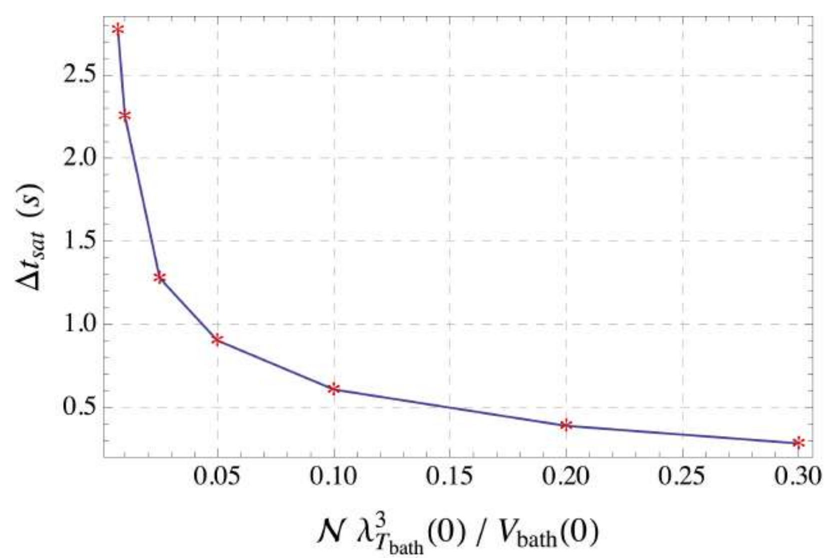
# Variation with phase space density

$$V_{\mathrm{bath}}(0)/V_{\mathrm{dimple}} = 5000$$

$$\beta(0)\,\varepsilon_{\mathrm{dimple}} \approx 15.6$$



# Three-body loss

- A collision of 3 Rb atoms can create a $\mathrm{Rb}_2^*$ molecule in excited vibrational state.
- The released energy causes both the atom and the molecule to escape.
- Rate of such processes at position $\vec{r}$

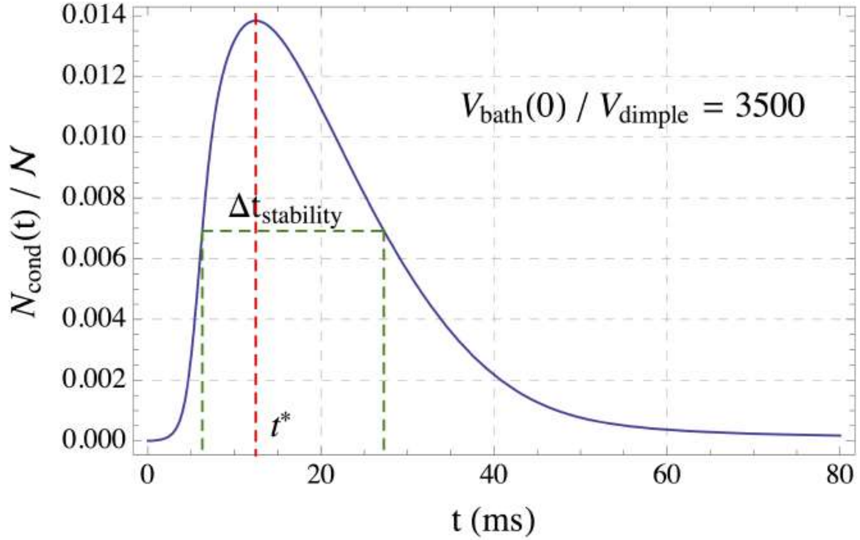
$$\propto n_0^3(\vec{r}) + 9\, n_0^2(\vec{r})\, n_{\rm th}(\vec{r}) + 18\, n_0(\vec{r})\, n_{\rm th}^2(\vec{r}) + 6\, n_{\rm th}^3(\vec{r})$$

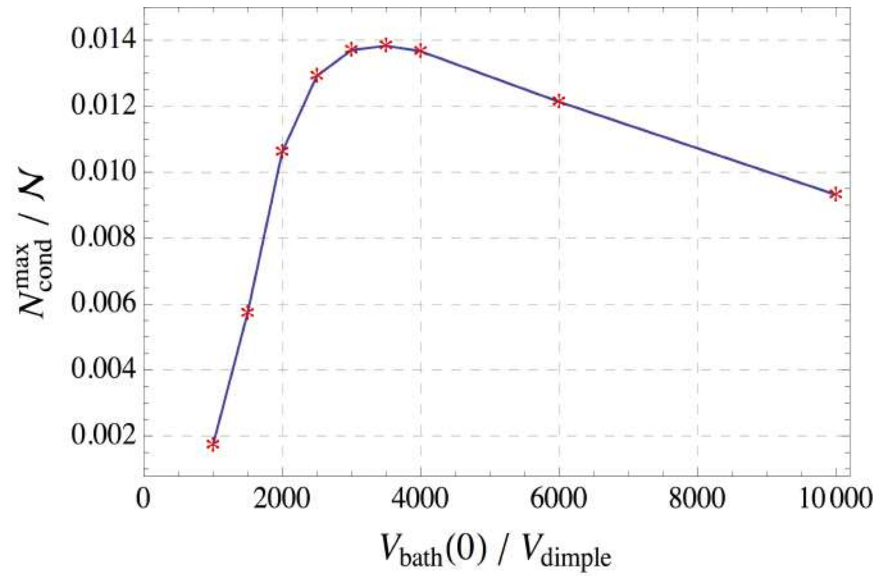
  where $n_0(\vec{r})$ and $n_{\rm th}(\vec{r})$ density of condensed and "thermal" particles.
- This contributes terms like $-\gamma\, n^2(\vec{r})$ , ($\gamma \approx 10^{-29} cm^6/s$) which destabilize condensates at high density.

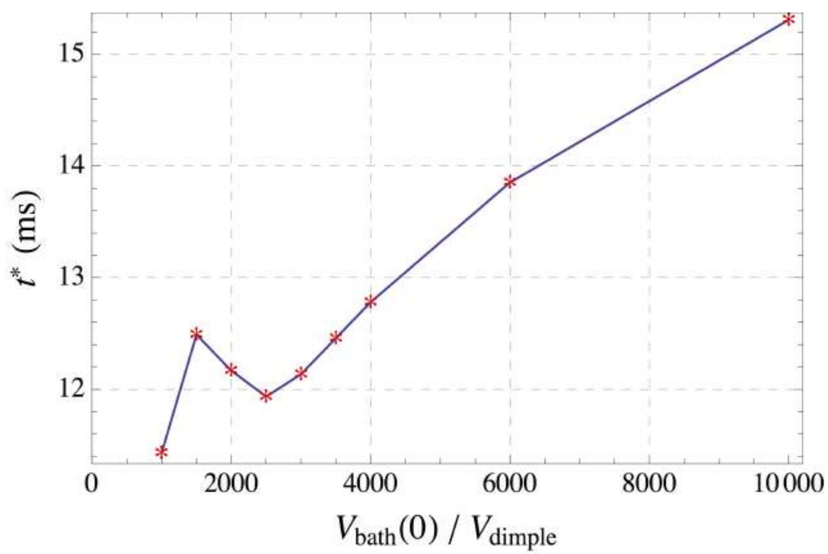
# Effect of three-body loss on condensate fraction

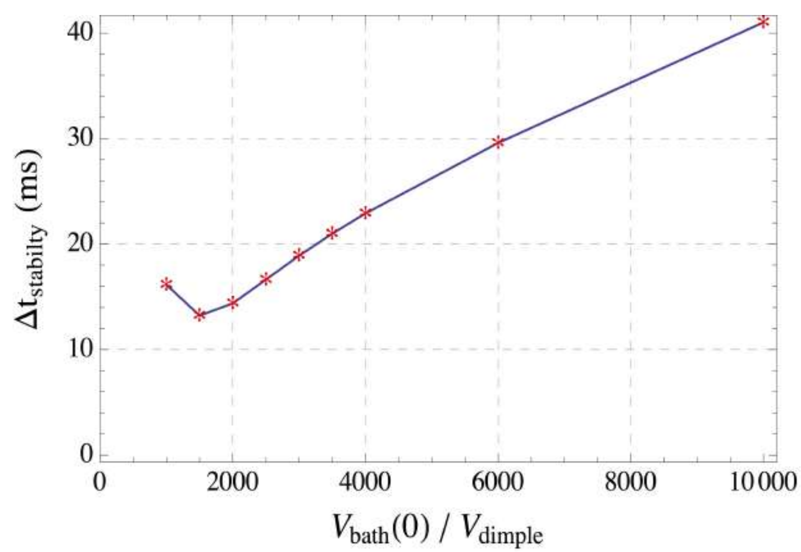
$$\rho_{\mathrm{bath}}(0)\lambda_T^3(0) = e^{-3}$$

$$\beta(0)\ \varepsilon_{\mathrm{dimple}} \approx 15.6$$



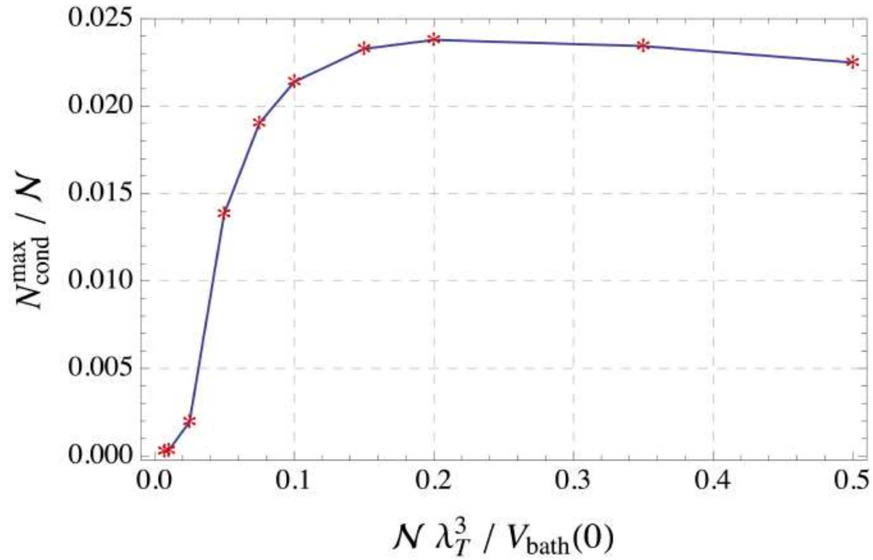



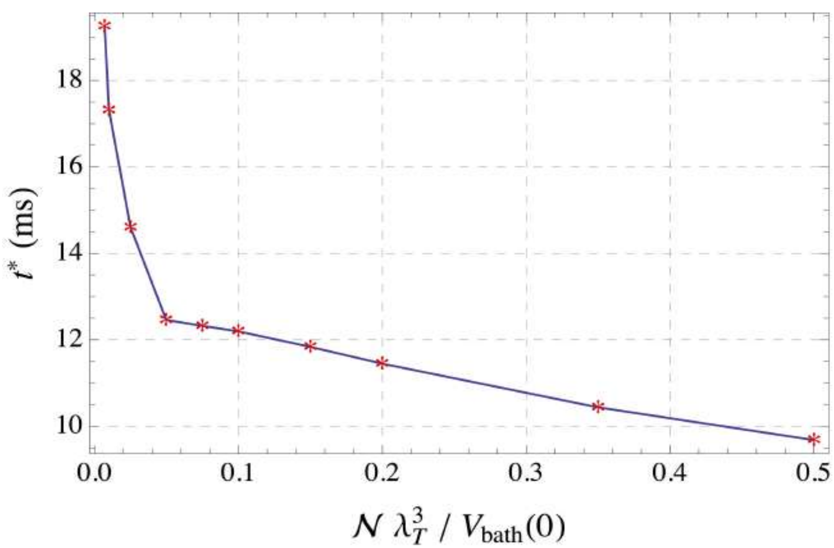



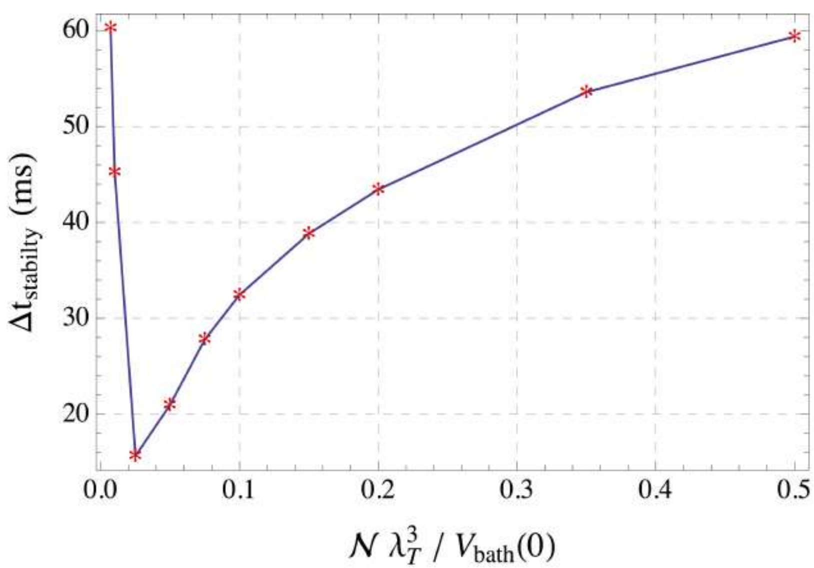



# Effect of three-body loss on condensate fraction

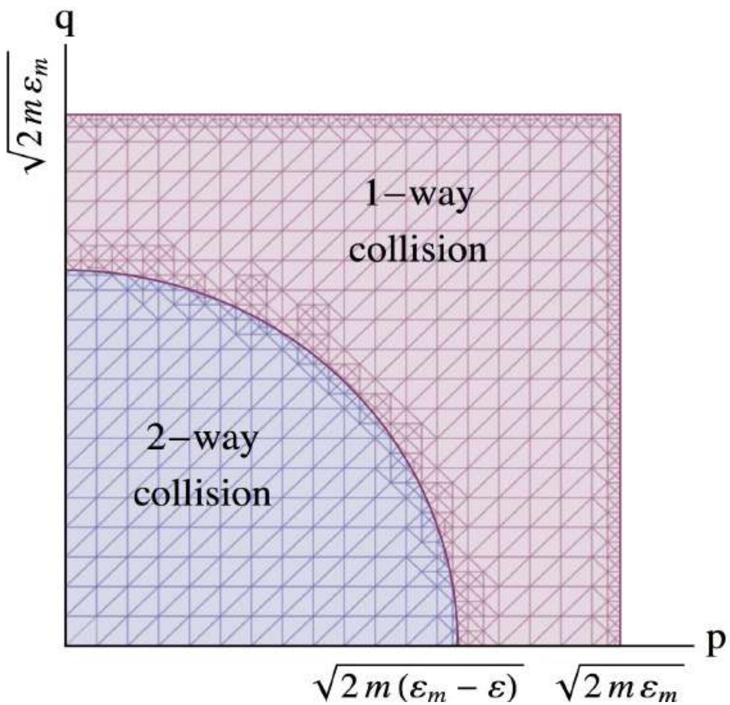
$$V_{\rm bath}(0)/V_{\rm dimple} = 3500$$
$$\beta(0)\ \varepsilon_{\rm dimple} \approx 15.6$$

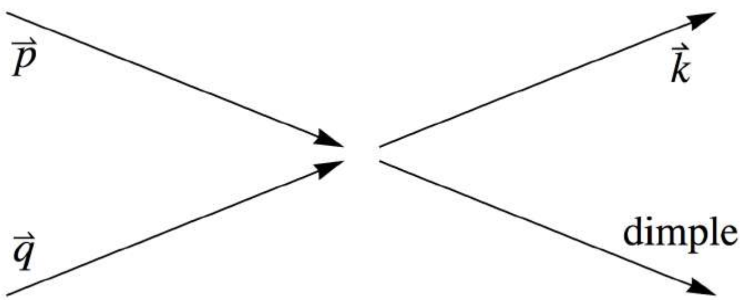


# Effect of finite trap depth



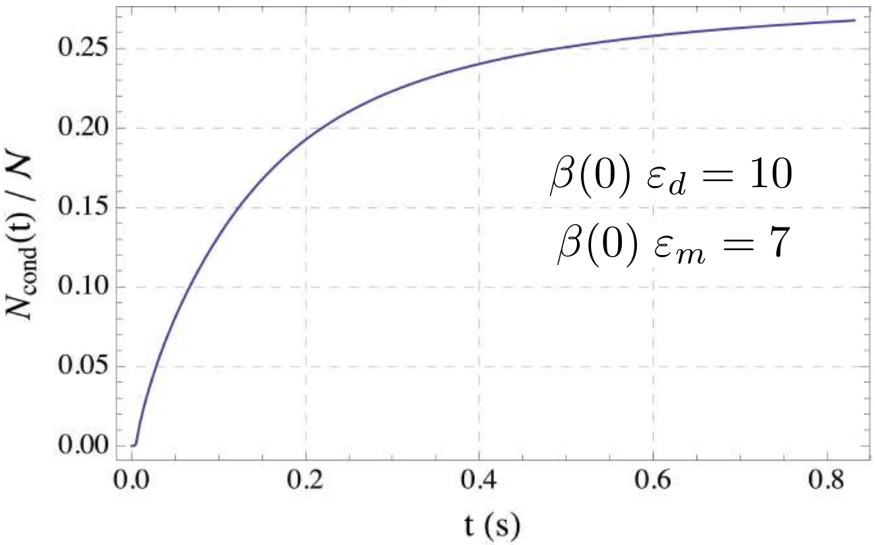



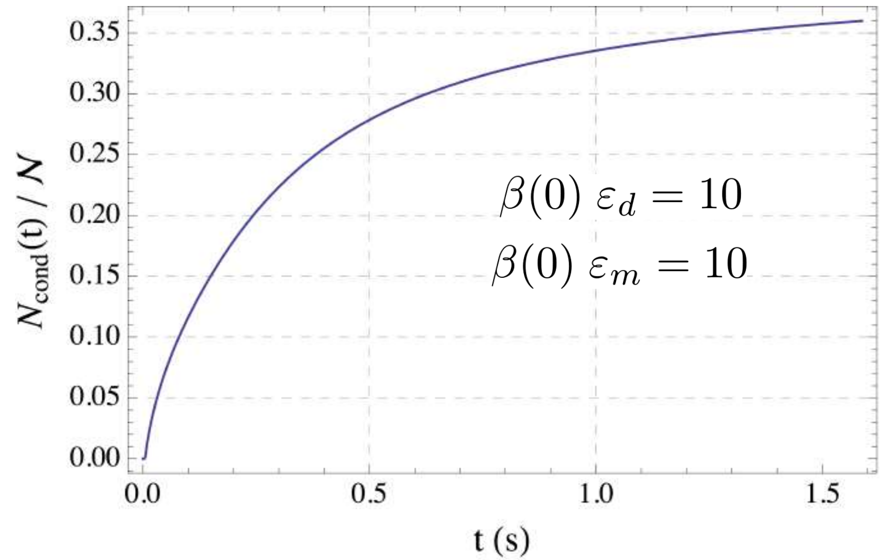

- 2-way collisions: no particle is lost
- 1-way collisions: one particle goes to the dimple and the other is lost
  => energy lost from bath = $\dfrac{(p^2+q^2)}{2m}$
- This causes cooling, resulting in higher condensate fraction
- For $\varepsilon_m \gg \varepsilon$, 1-way collisions become negligible

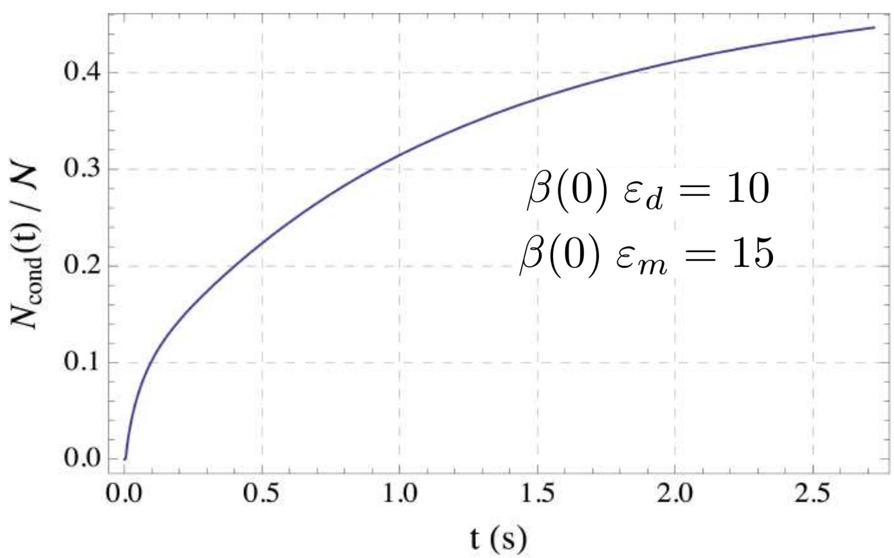
# Plots for finite trap depth (without 3 body loss)

$$\rho_{\text{bath}}(0)\lambda_T^3(0) = e^{-3}$$

$$V_{\text{bath}}(0)/V_{\text{dimple}} = 5000$$



# Thank you!

Questions?

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