FAQs on Prelim

(Dated: September 30, 2015)

Questions :

- **When does a charge distribution behave like a point charge?**

  I. When the charge distribution is **spherically symmetric**. Suppose you have a distribution which is spherically symmetric about a point P, i.e., the charge density depends only on the distance from P and not on the direction. Say you want to find the electric field at some point Q. Then you can join P to Q by a radius r, and draw a sphere centered at P. The field at Q would be the same as if the total charge within the sphere you drew were concentrated as a point charge at P. See, e.g., Q #4 in PS4.

  This results from a direct application of Gauss' Law, and has **nothing to do** with whether you have a conductor or an insulator. So, when in doubt, **use Gauss' Law**.

  II. If the distribution is finite (always true in the real world), and has a total charge of Q, then **far away** from the distribution, the field is like that of a point charge Q.

- **When can I use the superposition principle?**

  Always! The total field (or potential) at a point due to a set of charges will always be the sum of the fields (or potentials) produced by each charge individually. It **doesn't matter** whether you have some material (conductor or insulator) intervening the charges (see, e.g., Q #1 in PS4). Also, if you have a hole in some charge distribution, you can treat the hole as a sum of equal amounts of positive and negative charges (see, e.g., Q #4 in PS5).

- **σ/ϵ_0 or $\sigma/(2\epsilon_0)$?**

  $\sigma/(2\epsilon_0)$ is the magnitude of the electric field produced by a **large sheet of charge** (with charge density σ) on either sides. Whereas, σ/ϵ_0 is the magnitude of the **total** electric field at a point just outside a conducting surface, where the surface has a local charge density σ. This total field results from the contribution of **all** charges present, near or far. However, rather astonishingly, the total field depends only on the local surface charge density σ.

- **Is the electric field given by $\vec{E} = (kq/r^3)\vec{r}$?**

  Only true for **point charges**. It gives the field produced by a point charge q at a distance r. The vector $\vec{r}$ **always** points away from q (irrespective of whether q is +ve or -ve). The formula is also valid when you're considering a small element of a charge distribution (to set up an integral), or when you know that the charge distribution behaves like a point charge.

  Note : for a point charge, the field falls of as $1/r^2$. For a cylindrical (or line) charge distribution, it falls off as $1/r$. For a sheet of charge, it doesn't fall off at all. For a dipole, it falls off as $1/r^3$.

- **Potential energy vs potential**

  They are **different**. Potential energy is the amount of work done (by you) to put the system together. Whereas, potential (or voltage) is a function of space (a scalar field) produced by a collection of charges, much like the electric field (except scalar). When you move a test charge q from point a to point b, you do a work $W = q(V_b - V_a) = -q\int_a^b \vec{E}.d\vec{l}$, and thus increase the potential energy of the system by the same amount. This work done is **path-independent** (and equals minus the work done by the electric field).

- **When can I find $\vec{E}$ from V?**

  $\vec{E}$ is given by the rate of change of V as a function of space, $\vec{E} = -\vec{\nabla}V$. Thus, you must know how the potential is changing with the spatial co-ordinates to find $\vec{E}$. In other words, you need the **functional form** of V in terms of the co-ordinates. Knowing V at some particular point doesn't say anything about $\vec{E}$.

- **There is a hole in a conducting material. Does the conductor provide electrostatic shielding?**

  Yes. Suppose there are charges placed both inside the hole, as well as outside the conductor. Then charges on the **outer surface** of the conductor completely **shields the inside** (including the hole) from the influence of the outside charges (by canceling out their fields). Similarly, charges on the **inner surface** surrounding the hole completely **shields the outside** from the influence of charges in the hole.

- **Can I write $\oint \vec{E}.d\vec{A} = EA$ in Gauss' Law?**

  Only when the electric field has the same magnitude and is parallel to $d\vec{A}$ at all points on the Gaussian surface. This would be true when the charge distribution has some **symmetry** (e.g., a uniformly charged sphere or cylinder). However, even charges **outside** the Gaussian surface needs to have the same symmetry, since $\vec{E}$ denotes the **total field**, due to charges both inside and outside the surface.

  Note : Gauss' Law is **always** valid (though not always helpful). And although $\vec{E}$ has contributions from both inside and outside the surface, $\oint \vec{E}.d\vec{A}$ only depends on the total charge inside the surface (Q_{encl}).

Other points to note :

- Capacitors connected in **series** have the **same charge**, but not necessarily the same voltage difference. Capacitors connected in **parallel** have the **same voltage** difference, but not necessarily the same charge.
- When a **battery is connected** between the plates of a capacitor, it maintains the **same voltage** difference. On the other hand, when it is disconnected, the charges on the plates have nowhere to go, and stay constant.
- The potential energy of a collection of point charges is $U = (1/2)\sum_{ij} kq_iq_j/r_{ij}$. For a continuous charge distribution, this becomes $(1/2)\int V dq$, where the integral is over all space. For the special case of a capacitor, this reduces to $Q^2/(2C)$.
- For a conductor, (i) $\vec{E} = \vec{0}$ **inside**, (ii) all points have the **same potential**, (iii) just outside the surface, the **total** electric field is **perpendicular** to the surface, and equals σ/ϵ_0 where σ denotes the local charge density.
- **Dimensional analysis** provides an easy and useful way to check your answers (and one of the tools we'll use to detect wrong answers while grading!).
- Force, electric field, torque etc. are **vectors** - give both magnitude and direction. When you add vectors, take into account their directions. Don't write $\vec{E} = 5$ N/C, as both sides need to be either scalars or vectors.
- Don't forget to write **units** whenever you're giving a numerical answer.
- If you draw an arc at radius r, which subtends an angle $d\theta$ at the origin, the arc length is $rd\theta$, not $d\theta$.
- $V_{b\leftarrow a} \equiv V_{ba} = V_b - V_a$.