Conceptual problems on (introductory) Special Relativity

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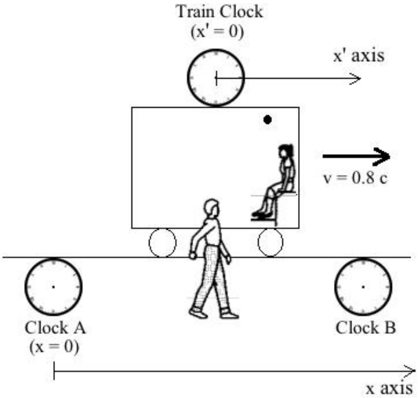
Alice throws a ball in a train

time dilation, proper time,
relativity of simultaneity, invariance of coincidences

It all hangs together by a tight-knit logic

Alice is traveling in a train at a speed $v = 0.8c$ ($\gamma = 5/3$) relative to Bob who is standing on the ground. Alice throws a ball upward as she passes by Clock A of the ground frame. When she catches the ball, she passes by Clock B of the ground frame. She measures that the ball was in air for $\Delta t' = 3$ s.



What time duration does Bob measure?

(a) 3 s

(b) 5 s

(c) 1.8 s

(d) None

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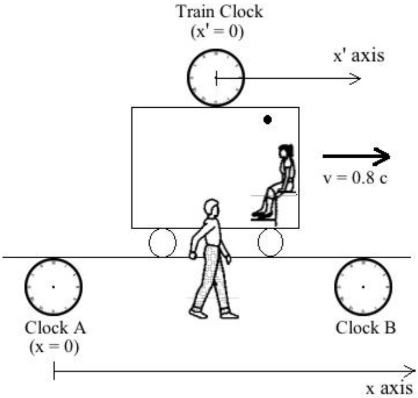
She measures that the ball was in air for $\Delta t' = 3$ s.



According to Bob, how much time did any clock on the train advance by while the ball was in air?

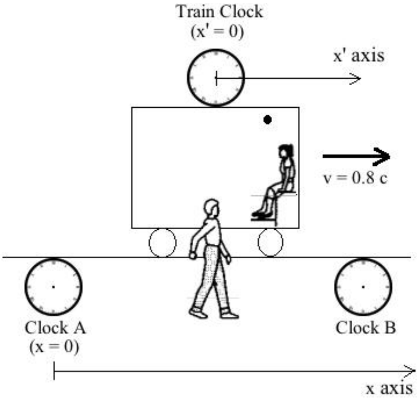
(a) 3 s

(b) 5 s

(c) 1.8 s

(d) 25/3 s

Alice is traveling in a train at a speed $v = 0.8c$ ($\gamma = 5/3$) relative to Bob . . . . She measures that the ball was in air for $\Delta t' = 3$ s.



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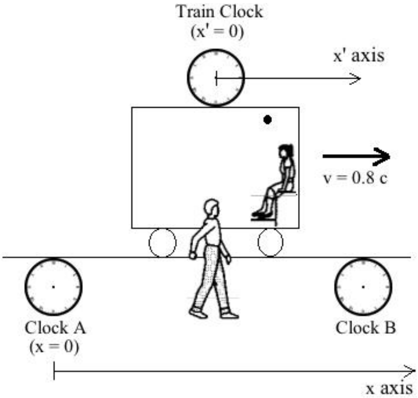

Who measures the *proper time* interval?

(a) Alice

(b) Bob

(c) None of them

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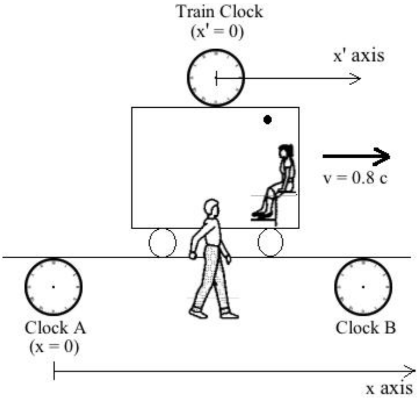


Bob measured that both Clock A and Train Clock were reading 0 when Alice threw the ball. Then, as measured by Alice,

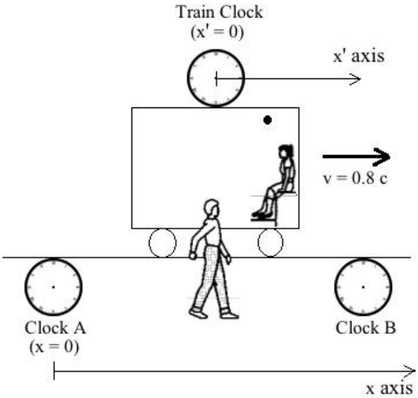
(a) Clock A was ahead of Train Clock.

(b) Clock A was behind Train Clock.

(c) Both were reading 0.

(d) Both were reading some other time.

Alice is traveling in a train at a speed $v = 0.8c$ ($\gamma = 5/3$) relative to Bob .... She measures that the ball was in air for $\Delta t' = 3$ s. Clock A and Train Clock were both reading 0 when Alice threw the ball.



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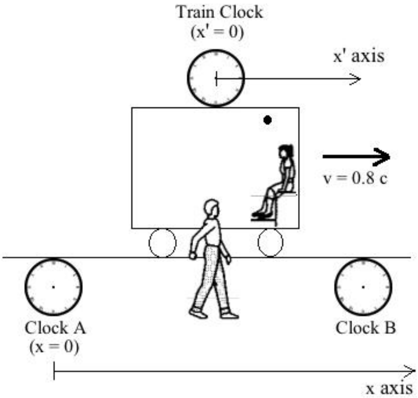

According to Alice, what was Clock B reading when she threw the ball?

Hint : The ball was in the air for 5 s, as measured by Bob.

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(b) 2 s

(c) 1.8 s

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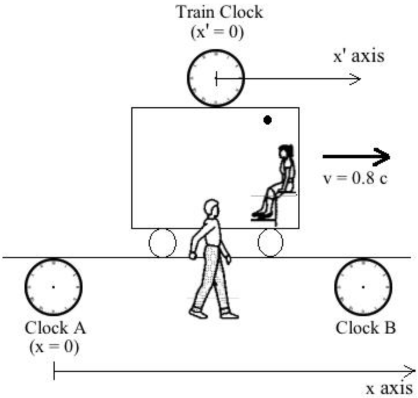

According to Alice, what was Clock B reading when she caught the ball?

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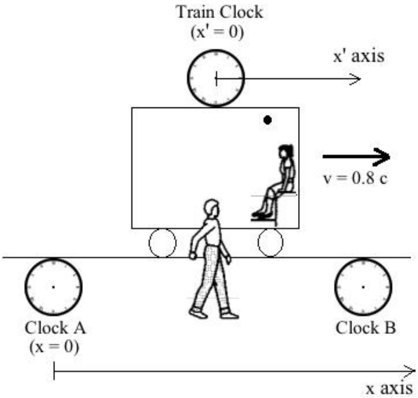

According to Bob, what was Clock B reading when Alice caught the ball?

(The ball was in the air for 5 s, as measured by Bob.)

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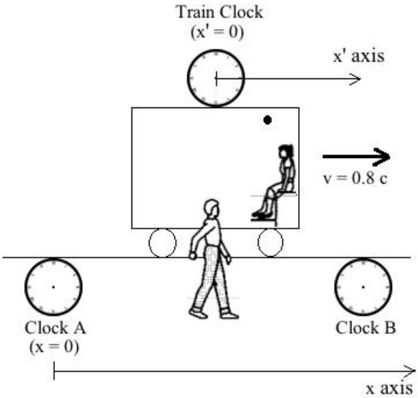

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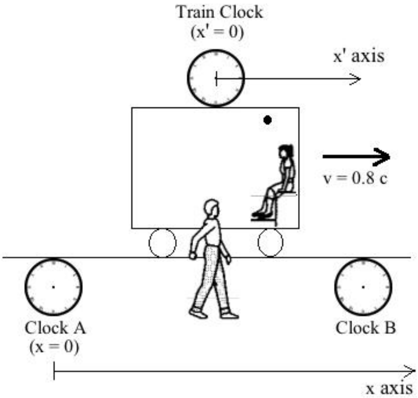

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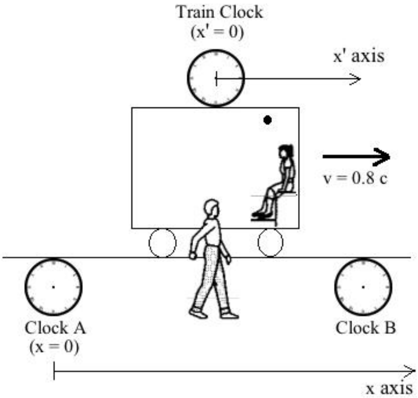

According to Alice, what was Clock A reading when she caught the ball?

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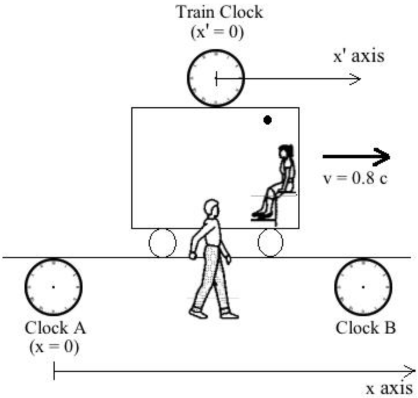

According to Bob, what was Clock A reading when Alice caught the ball?

(a) 3 s

(b) 5 s

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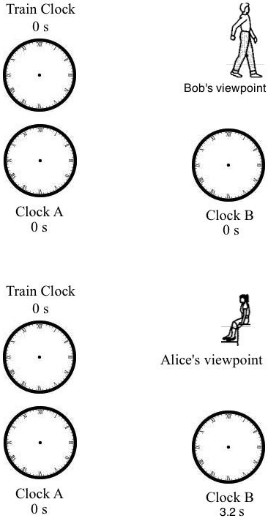
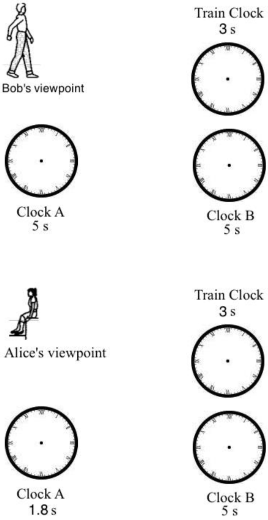
Alice throws the ball



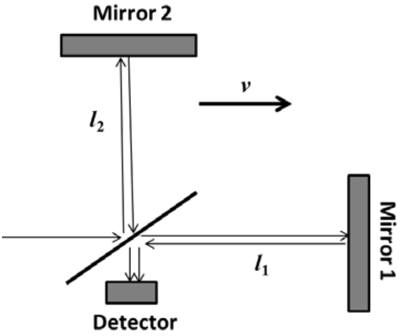
Alice catches the ball



Michelson-Morley experiment viewed from space

length contraction, time dilation,
invariance of coincidences

You are viewing the Michelson-Morley experiment being conducted on Earth from outer space. Both mirrors are at a distance L from the beam-splitter, as measured on Earth. However, Earth is moving past you at speed v.



What are the distances, as measured by you? ($\gamma = 1/\sqrt{1 - v^2/c^2}$)

(a) $l_1 = L, l_2 = L$

(b) $l_1 = L/\gamma, l_2 = L/\gamma$

(c) $l_1 = L/\gamma, l_2 = L$

(d) $l_1 = L, l_2 = L/\gamma$

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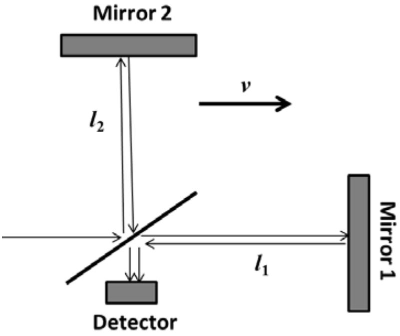

As measured by you, how much time does it take light to go from the beam-splitter to Mirror 1 and back?

(a) $2L/(\gamma c)$

(b) $2\gamma L/c$

(c) $2L/c$

(d) $2\gamma^2 L/c$

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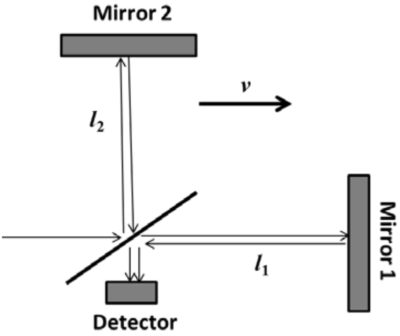


As measured by you, how much time does it take light to go from the beam-splitter to Mirror 2 and back?

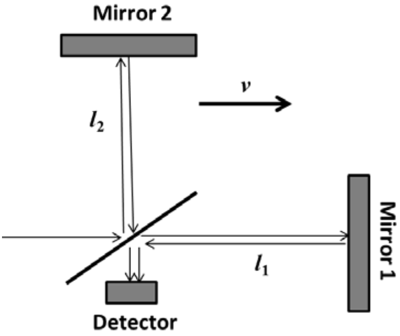
(a) $2L/(\gamma c)$

(b) $2\gamma L/c$

(c) $2L/c$

(d) $2\gamma^2 L/c$

You are viewing the Michelson-Morley experiment being conducted on Earth from outer space. Both mirrors are at a distance L from the beam-splitter, as measured on Earth. However, Earth is moving past you at speed v.



As per the time intervals measured by you, would you expect to see any interference pattern in the detector?

(a) Yes

(b) No

(c) Maybe??

Traveling long distances in one lifetime

time dilation, length contraction

In pursuit of alien life, you decide to go to the planet Kepler-442b which is about 1000 light-years from Earth. However, you cannot stay alive for more than 100 years in your spaceship. What minimum speed do you need to travel at, to reach the planet in your lifetime?

(a) $\sqrt{100/101}\,c$

(b) $\sqrt{99/100}\,c$

(c) c

(d) There is no such speed.

If you could travel at arbitrarily close to the speed of light (and had unlimited food supply), would you be able to circumnavigate the entire Milky Way in your lifetime, i.e., 100 years?

(a) Yes, provided I don't get trapped in a black hole!

(b) No way!

Reality of length contraction

Bell's thought experiment

(click to visit Wikipedia page on Bell's paradox)

Two spaceships are initially at rest in the ground frame, connected by a taut string of length L_0. At $t = 0$ in the ground frame, both spaceships simultaneously start moving to the right with the same constant acceleration a.

What happens to the string, as seen (measured) from the ground frame?

(a) It becomes shorter and shorter until it shrinks to zero.

(b) It becomes shorter and shorter, but approaches a finite length.

(c) It always retains the same length.

(d) It retains the same length, until it breaks due to increased tension.

Two spaceships are initially at rest in the ground frame, connected by a taut string of length L_0. At $t = 0$ in the ground frame, both spaceships simultaneously start moving to the right with the same constant acceleration a.

What is observed from an inertial frame moving at a speed close to c to the right relative to the ground?

(a) Both spaceships decelerate simultaneously, their separation increases, the string becomes longer, and breaks eventually.

(b) Both spaceships decelerate simultaneously, their separation remains the same, and the string becomes slack.

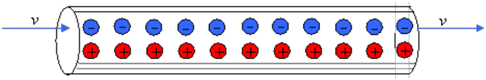
(c) The spaceship to the right of the figure decelerates much earlier, the string is stretched, and eventually breaks.

(d) The spaceship to the right of the figure decelerates earlier, the string is stretched, but does not break.

Charges in a current-carrying wire

length contraction, conservation of charge

In a current-carrying wire, electrons drift at speed v, while protons are at rest. However, in any given length, measured in the wire's rest frame, there are same number of protons and electrons. So the wire is electrically neutral.



As seen from a frame also moving to the right at speed v,

(a) The electrons are farther apart from each other than protons, so the wire as a whole is positively charged!

(b) We still have the same number of electrons and protons, so any segment of the wire is still electrically neutral.

(c) The electrons are farther apart from each other than protons in the segment shown, which is positively charged, but the wire as a whole is neutral.

(d) The protons are farther apart from each other than electrons in the segment shown, which is negatively charged, but the wire as a whole is neutral.

Universality of time dilation

principle of relativity

Suppose you are traveling in a high-speed train relative to the ground. As measured by a ground observer, the clocks on your train are ticking slower. Which of the following is true about this time dilation effect?

(a) It only applies for light-clocks.

(b) It applies universally to all physical, chemical, and biological processes, if the principle of relativity is to hold.

(c) It applies universally to all physical, chemical, and biological processes, because of some other unknown reason.

(d) Its just an artifact of measurement, and not actually real.

Trans-galactic phone call

causality, speed limit, timelike events

On 25th December 2200, You saw John making a trans-galactic phone call to his assistant Mo in Barnard's star, which is about 6 light-years away from Earth. 10 years later, you got to know that there was a big explosion on Barnard's star on 30th August, 2204. Which of the following can you conclude?

(a) John could not have influenced that explosion in any way.

(b) John could have instructed Mo to make that explosion.

(c) It must have been John who made it happen! I knew he was a monster!

(d) Not enough info to arrive at any conclusion.

Event in relativity

Which of the following qualifies as an “event” in relativity?

(a) Rio Olympics 2016!

(b) You taking a bus from your home to campus.

(c) You catching a frisbee on Arts quad at 5 pm yesterday.

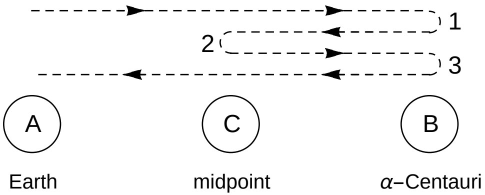
(d) You spending 4 years at Cornell.

(e) All of the above.

Twin paradox with a twist

relativity of simultaneity, time dilation,
invariance of coincidences, causality

You embark on a round-trip journey to α-Centauri which is 4 light-years away from Earth. Along the way, you meet three clocks which are at rest relative to the Earth : starting from Clock A on Earth, you reach Clock B on α-Centauri, then back to Clock C at the midpoint, back again to Clock B, and finally back to Earth. You maintain a speed of $0.8c$ ($\gamma = 5/3$) throughout.



According to you, what is the reading on Clock B just before reaching the turning point 1?

(a) 5 years

(b) 3 years

(c) 1.8 years

(d) 8.2 years

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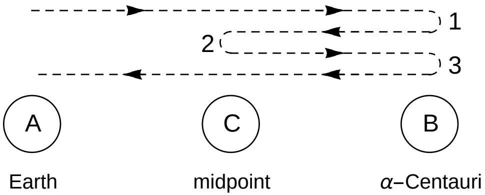

According to you, what is the reading on Clock A just before reaching the turning point 1?

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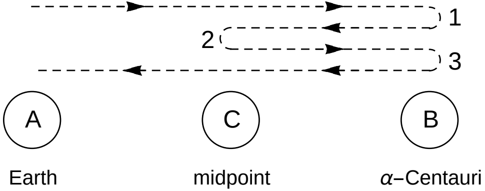

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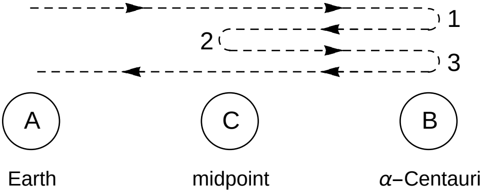

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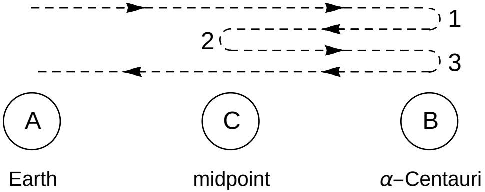

According to you, what is the reading on Clock C just before reaching the turning point 2?

(a) 5.9 years

(b) 9.1 years

(c) 7.5 years

(d) None of these.

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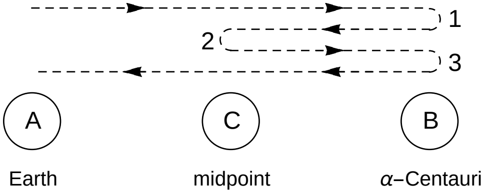

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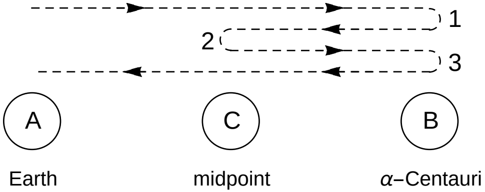

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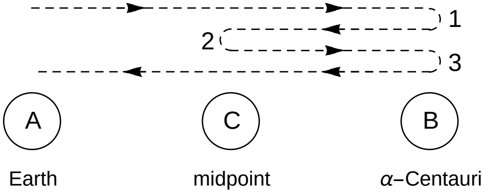


According to you, what is the reading on Clock A just after reaching the turning point 2?

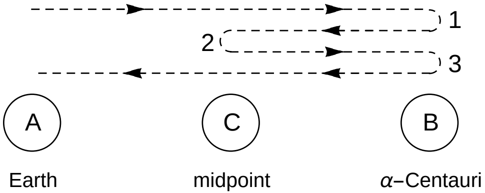
(a) 5.9 years

(b) 9.1 years

(c) 7.5 years

(d) None of these.

You embark on a round-trip journey to α-Centauri which is 4 light-years away from Earth . . . . You maintain a speed of $0.8c$ ($\gamma = 5/3$) throughout.



According to you, the reading on Clock A changed from 9.1 years to 5.9 years as you made the swift u-turn near Clock C! Is causality violated here?

(a) Yes!

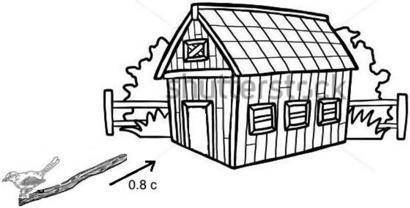
(b) No!

(c) This is crazy!

Barn, stick, bird, and the monster

invariance of events, universal speed limit, relativity of simultaneity, length contraction

A stick is traveling at a speed of 0.8 c ($\gamma = 5/3$) toward a barn of the same rest length (ignore gravity). A bird is sitting on the tail end of the stick. Gremlin the monster has placed a bomb in the barn. He plans to shut the door of the barn as soon as the entire stick goes inside, thus trapping the bird. However, the bird will fly off if it detects any vibration in the stick (assume that the bird can fly off instantaneously). Does it survive?



(a) In the ground frame, the stick fits inside the barn, the bird is trapped, and dies. Whereas in the stick's rest frame, the bird flies off. Its a paradox!

(b) Since the ground frame is more fundamental, the bird will actually die :(

(c) Observers in both frames agree that the bird will fly away :)

(d) Observers in both frames agree that the bird will die :(

Seeing a moving digital clock

seeing vs measuring, blue shift

Suppose a digital clock on a platform is traveling toward you at speed 0.8 c. It flashes every second in its rest frame. How much time passes in your frame between two successive flashes?

(a) 1.25 s

(b) 0.8 s

(c) 0.5 s

(d) 2 s

Suppose a digital clock on a platform is traveling toward you at speed 0.6 c ($\gamma = 5/4$). It flashes every second in its rest frame. How much time passes in your frame between two successive flashes *reaching your eye*?

(a) 1.25 s

(b) 0.8 s

(c) 0.5 s

(d) 2 s

Particle decaying into a photon

conservation of energy and momentum

A particle of rest mass m traveling to the right at speed v decays into a single photon of energy E, also traveling to the right. Which of the following is true?

(a) $E = mc^2/\sqrt{1 - v^2/c^2}$

(b) $E = mvc/\sqrt{1 - v^2/c^2}$

(c) $E = mc^2$

(d) Such a process cannot occur if our laws are correct (why?).

Einstein's thought experiment

conservation of energy and momentum, Doppler effect

A particle sitting at rest on the ground emits two photons in opposite directions. Each photon has energy E in the ground frame. As measured from a train traveling to the right at speed v, the two photons have energies E_l and E_r, with l and r denoting "left" and "right" respectively. Then,

(a) $E_l + E_r = 2E$

(b) $E_l + E_r = 2\gamma E \qquad (\gamma = 1/\sqrt{1 - v^2/c^2})$

(c) $E_l + E_r = 2E/\gamma$

(d) None of these.

A particle sitting at rest on the ground emits two photons in opposite directions. Each photon has energy E in the ground frame. As measured from a train traveling to the right at speed v, the two photons have energies E_l and E_r, with l and r denoting "left" and "right" respectively. Then,

(a) $E_l - E_r = 2Ev/c$

(b) $E_l - E_r = 2\gamma Ev/c$

(c) $E_l - E_r = 2Ev/(\gamma c)$

(d) $E_l - E_r = 0$

A particle sitting at rest on the ground emits two photons in opposite directions. Each photon has energy E in the ground frame. As measured from a train traveling to the right at speed v, the two photons have energies E_l and E_r, with l and r denoting "left" and "right" respectively.

It is known that $E \propto 1/\lambda$ for photons, where λ is the wavelength. Then,

(a) $\lambda_l = \lambda\sqrt{\frac{1-v/c}{1+v/c}}\,, \quad \lambda_r = \lambda\sqrt{\frac{1+v/c}{1-v/c}}\,.$

(b) $\lambda_l = \lambda\sqrt{\frac{1+v/c}{1-v/c}}\,, \quad \lambda_r = \lambda\sqrt{\frac{1-v/c}{1+v/c}}\,.$

(c) $\lambda_l = \lambda\,, \quad \lambda_r = \lambda\,.$

(d) None of these.

Pair annihilation

conservation of momentum, center-of-momentum frame, invariance of the speed of light

An electron traveling along the $+x$ direction collides with a stationary positron. They annihilate each other, emitting two photons in the process. Assuming both photons travel along the x axis, what can we say about their directions?

(a) Both photons travel along $+x$.

(b) Both photons travel along $-x$.

(c) They travel in opposite directions.

(d) It depends on the speed of the electron - have to do the math.

Inelastic collision in relativity

rest mass, energy, heat, and sound

Two particles of rest mass m traveling with speed v in opposite directions collide with each other, and fuse together to form a particle of rest mass M. Treating the collision as *non-relativistic*, which of the following is true?

(a) $M = 2m$, and mv^2 amount of energy is lost in the collision in the form of heat and sound.

(b) $M > 2m$, and no energy is lost in the collision.

(c) $M < 2m$, and no energy is lost in the collision.

(d) $M = 2m$, and no energy is lost in the collision. Instead, the final particle travels at a non-zero speed.

Two particles of rest mass m traveling with speed v in opposite directions collide with each other, and fuse together to form a particle of rest mass M. Treating the collision as *relativistic*, which of the following is true?

(a) $M = 2m$, and mv^2 amount of energy is lost in the collision in the form of heat and sound.

(b) $M > 2m$, and no energy is lost in the collision.

(c) $M < 2m$, and no energy is lost in the collision.

(d) $M = 2m$, and no energy is lost in the collision. Instead, the final particle travels at a non-zero speed.

Two particles of rest mass m traveling with speed v in opposite directions collide with each other, and fuse together to form a particle of rest mass M. What happens to the heat and sound in the relativistic picture?

(a) The relativistic theory is incapable of describing heat or sound.

(b) Heat or sound is generated only if the colliding particles are traveling at non-relativistic speeds.

(c) The heat generated in the collision contributes to the increase in rest mass. To describe sound, one needs to consider the surrounding air molecules in the collision.

(d) Its all a big mystery!

Rest mass and energy in collisions

invariance vs conservation, non-relativistic vs relativistic

In a *non-relativistic* collision, the total rest mass of all particles is

(a) frame-invariant and conserved.

(b) frame-invariant, but not conserved.

(c) not frame-invariant, but conserved.

(d) neither frame-invariant nor conserved.

In a *relativistic* collision, the total rest mass of all particles is

(a) frame-invariant and conserved.

(b) frame-invariant, but not conserved.

(c) not frame-invariant, but conserved.

(d) neither frame-invariant nor conserved.

In a *non-relativistic elastic* collision, the total kinetic energy of all particles is

(a) frame-invariant and conserved.

(b) frame-invariant, but not conserved.

(c) not frame-invariant, but conserved.

(d) neither frame-invariant nor conserved.

In a *relativistic* collision, the total energy of all particles is

(a) frame-invariant and conserved.

(b) frame-invariant, but not conserved.

(c) not frame-invariant, but conserved.

(d) neither frame-invariant nor conserved.

Balancing a mirror by light

photon momentum, force exerted by light

Consider a 50 Watt light-bulb pointed up toward a horizontal mirror which reflects all the light back. How many such light-bulbs would you need to support the weight of the mirror, if it weighs 1 kg? (Take $g = 10$ m/s^2)

(a) 3×10^8

(b) 3×10^7

(c) Infinitely many!

(d) Just a few.

(e) None of these