Collective modes of a soliton train in a Fermi superfluid

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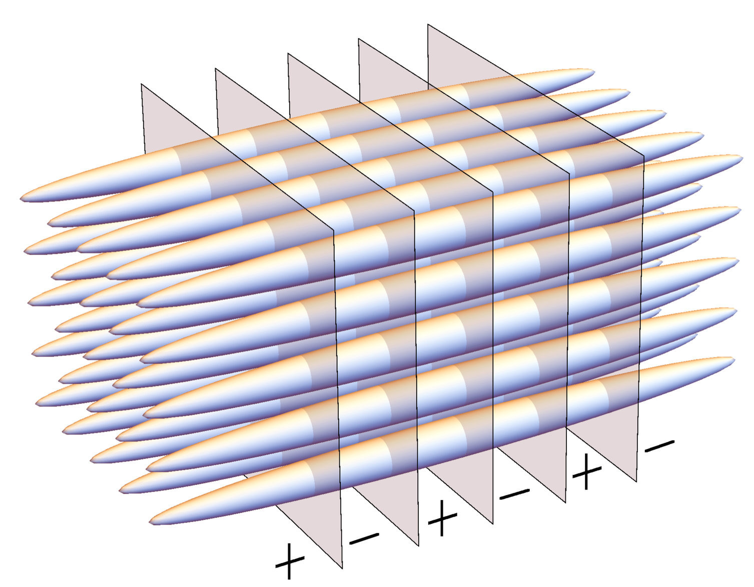
I. Motivation

Solitons :

- example of persistent non-linear structures, a common motif in many-body non-equilibrium dynamics.
- trains generated in quench experiments [1].
- next generation of phase-imprinting experiments will engineer soliton trains in Fermi superfluids [2].
- behavior and stability of these trains are not understood.

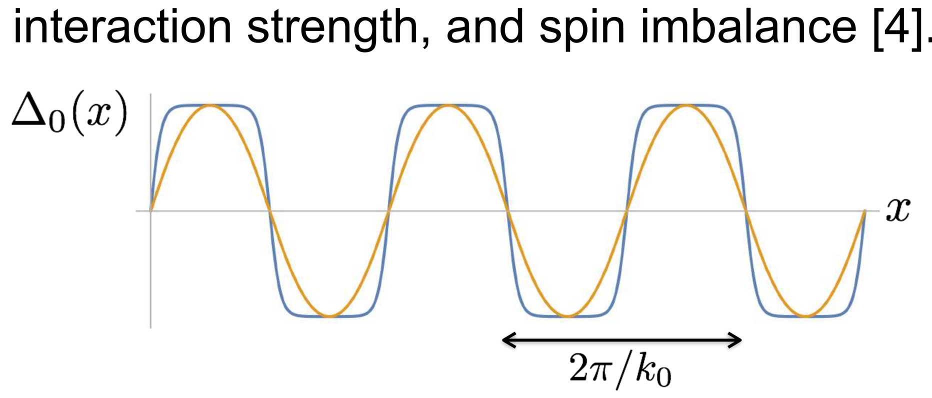
II. Possible experimental set-up



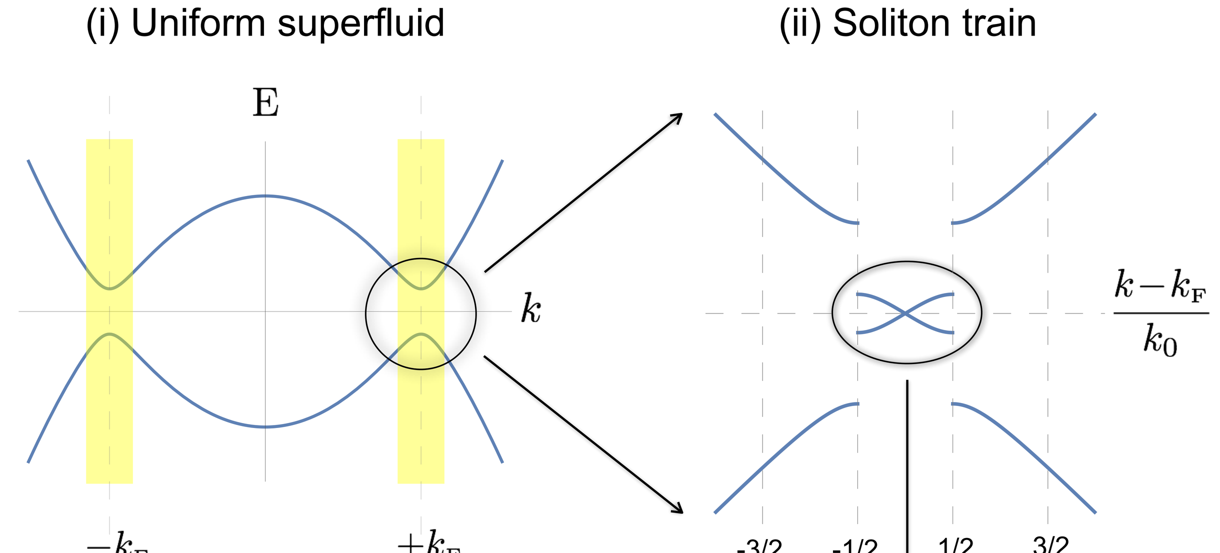
1. Create Fermi superfluid in array of tubes (no snake instability, long-range order [3]).
2. Phase imprint multiple solitons.
3. Study their collective motion.

III. Stationary solution

$\Delta_0(x) \propto \mathrm{sn}(x, \zeta)$, where ζ is set by the soliton spacing, interaction strength, and spin imbalance [4].



Single-particle spectrum :



Andreev bound states inside solitons

IV. Theoretical modeling of collective oscillations

- Self-consistent Bogoliubov de-Gennes (BdG) formalism.
- Extract collective modes from linearized dynamics of fluctuations about stationary solution.
- Weak coupling $\Longrightarrow$ amplitude and phase fluctuations of order parameter decouple.
- Characterized by three parameters :
  (i) k_0/k_F : wave-vector of the train
  (ii) $k_F a_{1D}$: interaction strength
  (iii) n_s : # of unpaired fermions per soliton (i.e., spin imbalance).

V. Results

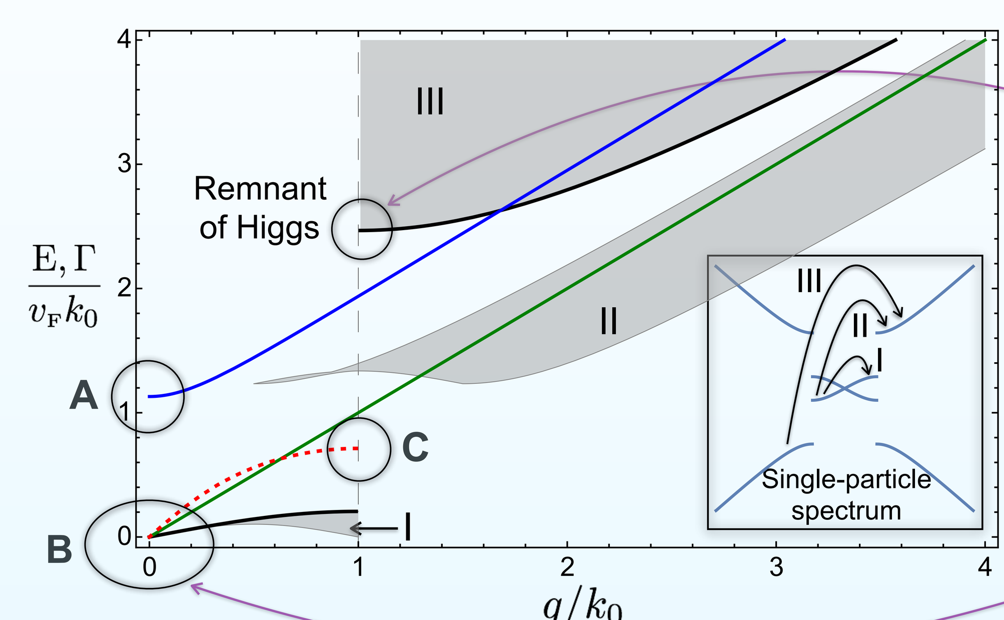
1. Spectrum contains: 2 Goldstone modes, "Higgs" mode, novel gapped modes describing oscillations of soliton cores, 3 disconnected continua, and a growing instability toward a uniform state.

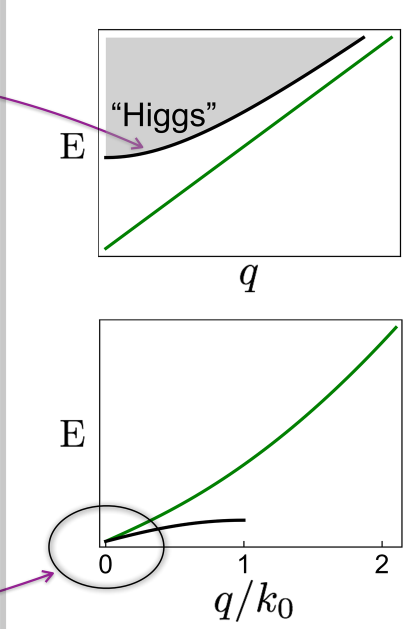
Collective mode spectrum of a soliton train without spin imbalance, for $k_0/k_F = 0.05$, $k_F a_{1D} = 2.6$.

Gray regions : particle-hole continua.

Solid curves : energy of stable modes.

Dotted curve : growth rate (Γ) of unstable mode.



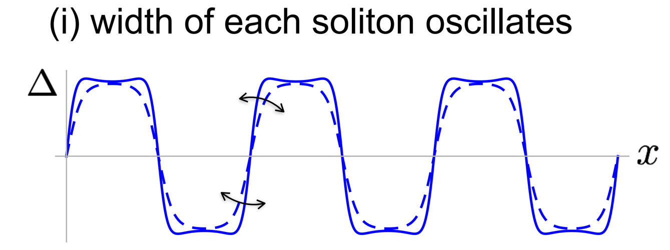


Collective modes of a uniform Fermi superfluid

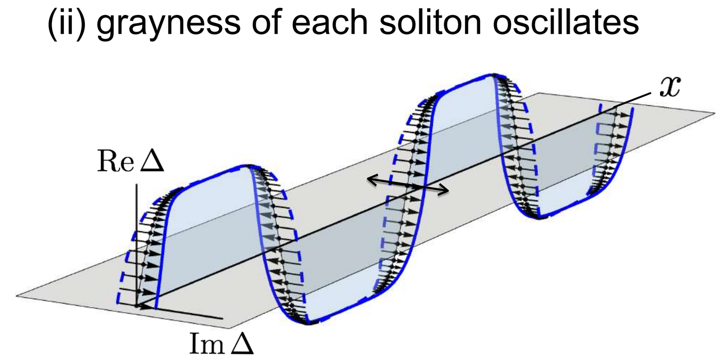
Collective modes of a soliton train in a Bose superfluid

A. Two-fold degenerate "core" modes

(i) width of each soliton oscillates

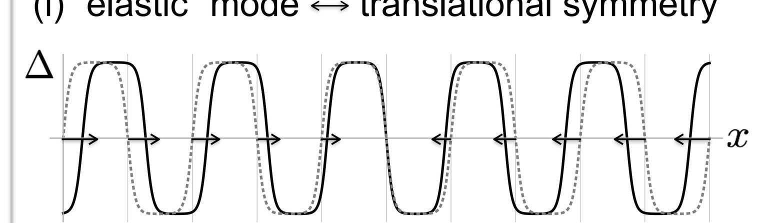


(ii) grayness of each soliton oscillates

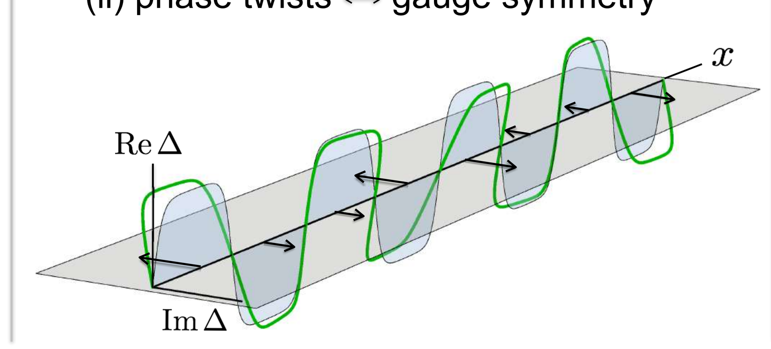


B. Two gapless Goldstone modes

(i) "elastic" mode ⟷ translational symmetry

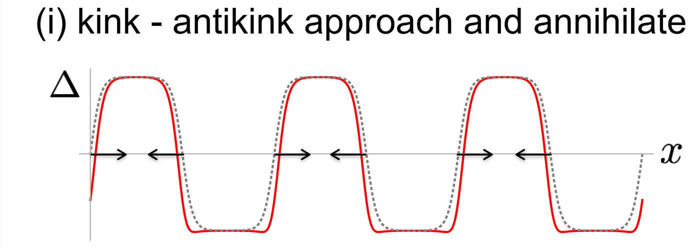


(ii) phase twists ⟷ gauge symmetry

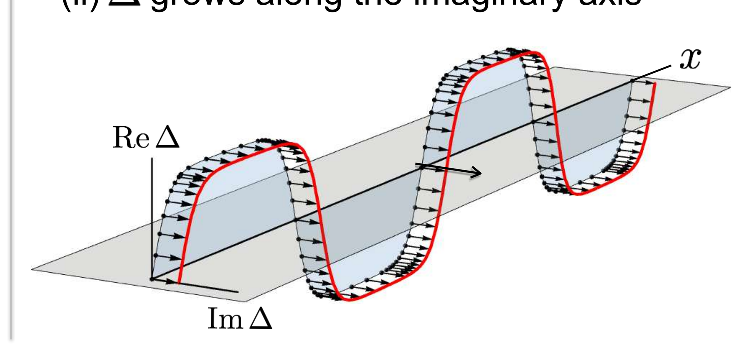


C. Two-fold degenerate unstable modes

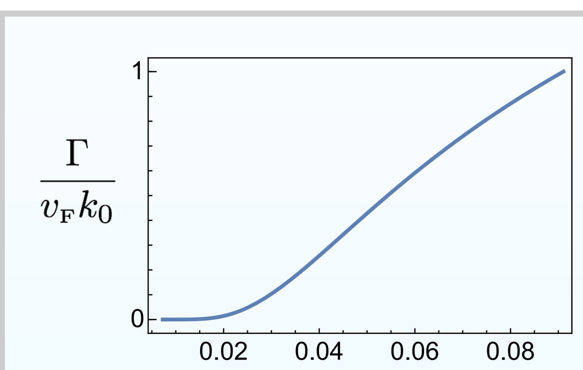
(i) kink - antikink approach and annihilate



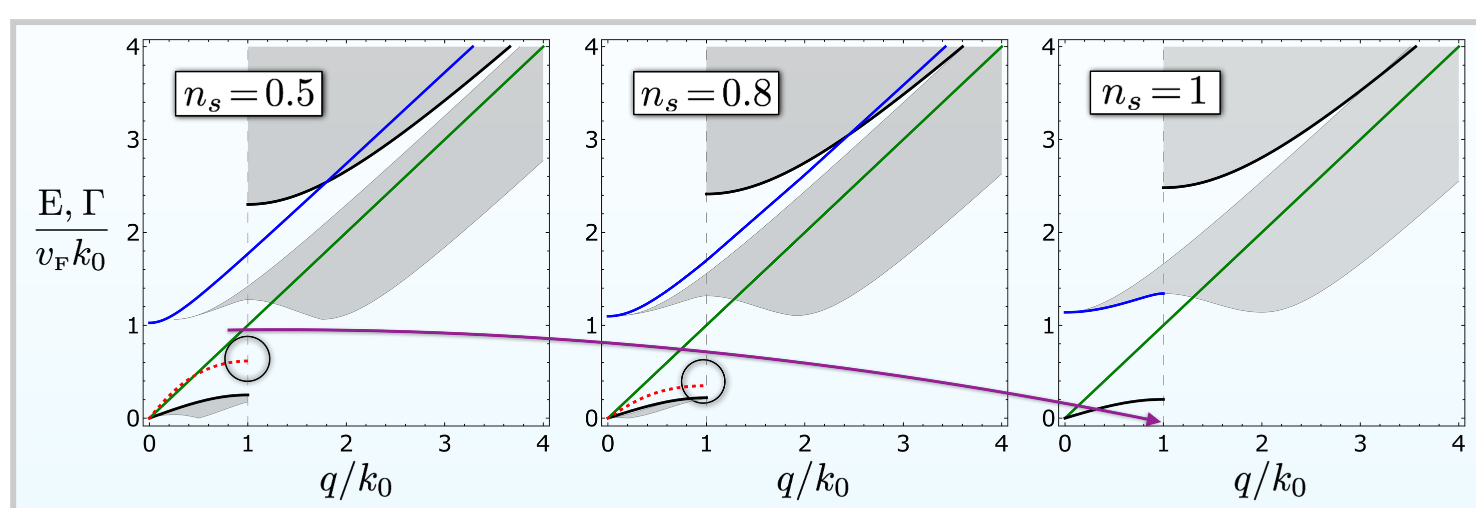
(ii) Δ grows along the imaginary axis



2. The train can be stabilized by creating solitons farther apart or filling them with unpaired fermions.



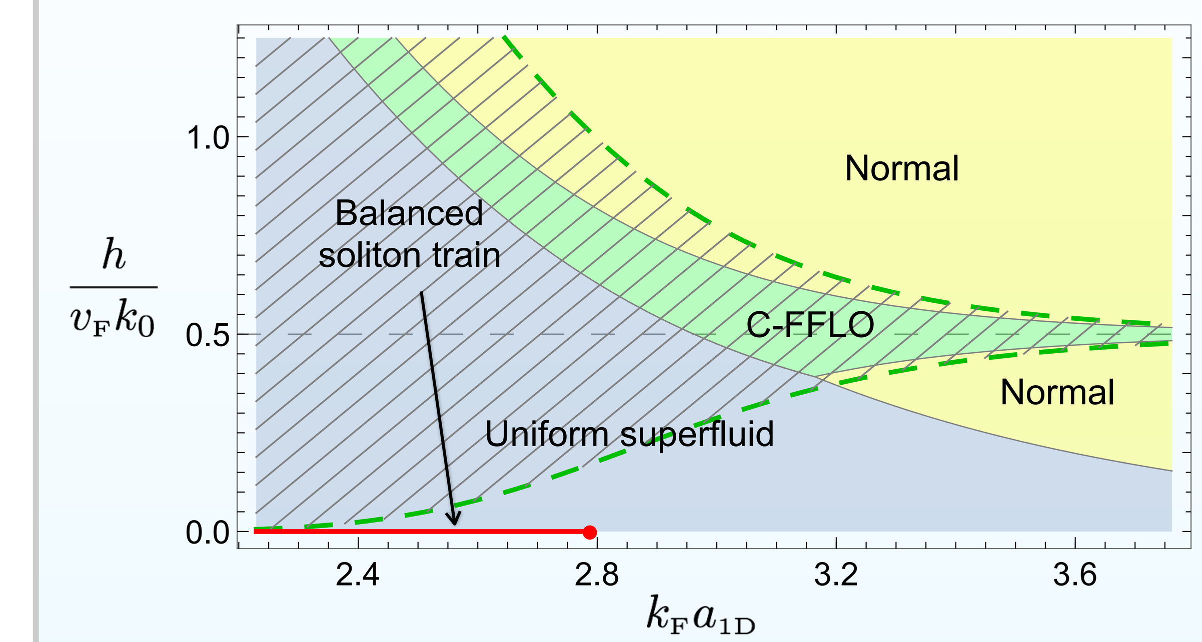
Instability rate vs inverse soliton spacing, for $k_F a_{1D} = 2.4$.



Variation with spin imbalance, for $k_0/k_F = 0.05$, $k_F a_{1D} = 2.65$.

VI. Results (cont'd)

3. The commensurate FFLO phase (C-FFLO), with one unpaired fermion per soliton, is dynamically stable, even when there are lower energy states available.



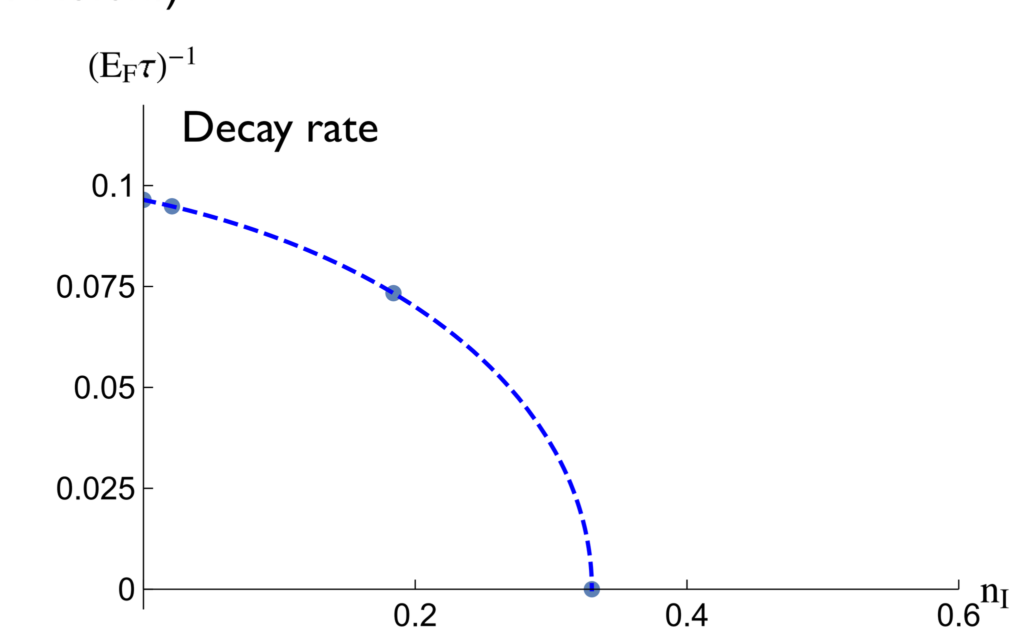
Phase diagram (with $k_0/k_F = 0.05$ for soliton trains) : Solid regions show lowest energy states. C-FFLO is dynamically stable throughout the hashed region.

h — chemical potential difference

VII. Conclusions

- Soliton train unstable: stabilized by increasing separation (also interaction strength), or adding imbalance.
- New modes : oscillations in width/grayness of solitons.
- C-FFLO phase dynamically stable even when not ground state — create with phase imprinting.

3D: imbalance helps stabilize against snake instability (M. Reichl)



Density of up-spins in domain wall
(measured in units of density far from domain wall)

1. Lamporesi *et. al.*, Nature Phys. 9, 656 (2013).
2. M. J. H. Ku *et. al.*, Phys Rev. Lett. 116, 045304 (2016).
3. Y. Liao *et. al.*, Nature 467, 567 (2010).
4. K. Machida *et. al.*, Phys. Rev. B 30, 122 (1984).