Thermalization in a Quasi-1D Quantum Gas

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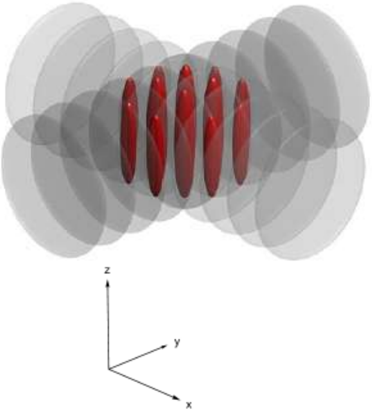
Physical System



- Atoms are captured in a 3D harmonic trap
- A deep optical lattice is turned on in the x-y plane
- Intertube coupling J is small
- Initial atom energies are small compared to the band gap

Two cases

Lattice turn on is slow compared to τ_{coll}

Lattice turn on is fast compared to τ_{coll}

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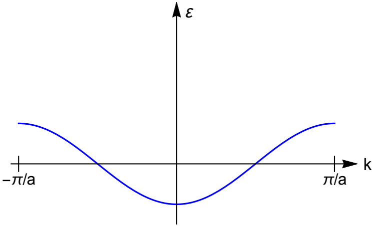
- Atoms are confined to the lowest band
- Model : Tight-binding



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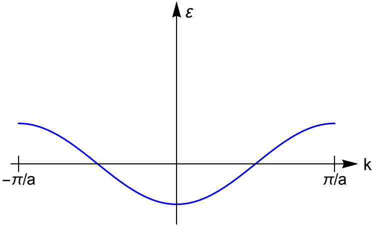
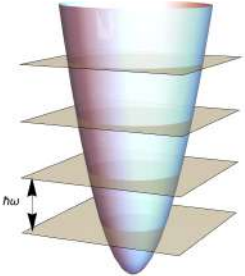
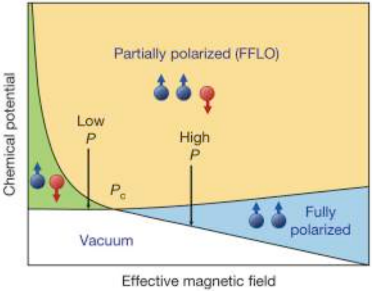


Lattice turn on is fast compared to τ_{coll}

- Atoms populate several higher bands
- Model : 2D harmonic well



Rice : Spin-polarized fermions (^{6}Li) Nature **467**, 567 (2010)



Measurements agree with thermodynamic calculations

Motivation : Apparent conflict between two experiments

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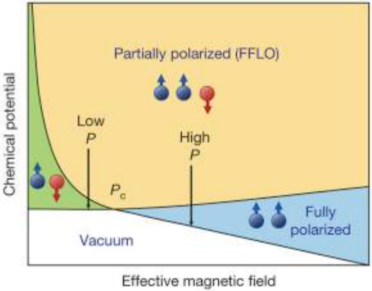
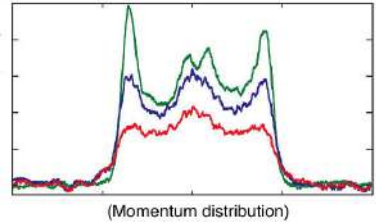


Measurements agree with thermodynamic calculations

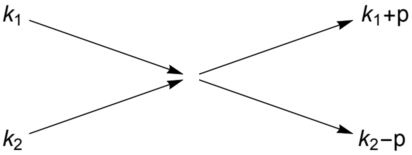
Penn State : Spinless bosons (^{87}Rb) Nature **440**, 900 (2006)



Atoms do *not* thermalize!

In both cases, lattice turn on is slow – dynamics in lowest band

Dynamics confined to the lowest band



$$(k_1 + p)^2 + (k_2 - p)^2 = k_1^2 + k_2^2 + \Delta\varepsilon_\perp \,, \qquad \Delta\varepsilon_\perp \sim J$$

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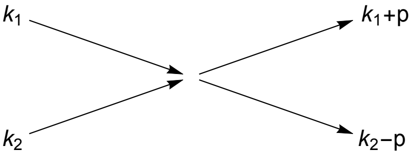

$$(k_1+p)^2+(k_2-p)^2=k_1^2+k_2^2+\Delta\varepsilon_\perp\,, \qquad \Delta\varepsilon_\perp\sim J$$

$$\dot{n}(k)=\int\!\!\int dpdq\; f(pq)\;\times$$

[f is defined in Appendix 1]

$$\Big[n(k+\tilde{J}^{\frac{1}{2}}p)n(k+\tilde{J}^{\frac{1}{2}}q)\big(1+\zeta n(k)\big)\big(1+\zeta n(k+\tilde{J}^{\frac{1}{2}}(p+q))\big)$$
$$-\big(1+\zeta n(k+\tilde{J}^{\frac{1}{2}}p)\big)\big(1+\zeta n(k+\tilde{J}^{\frac{1}{2}}q)\big)n(k)n(k+\tilde{J}^{\frac{1}{2}}(p+q))\Big],$$

where $\tilde{J}\equiv\pi^2(J/E_R)$, $\zeta=+1$ (bosons), -1 (fermions), 0 (classical)

Small $\tilde{J}$ expansion

$$\dot{n}(k) = \big(\dot{n}(k)\big)_{\text{cl}} + \big(\dot{n}(k)\big)_{\text{qu}} + \mathcal{O}(\tilde{J}^4 \ln \tilde{J}) \; ,$$

where

$$\big(\dot{n}(k)\big)_{\text{cl}} = \tilde{J}^2 \Big[\Big((3 - \ln \tilde{J}) I_2 - I_{2I} \Big) \mathcal{F}_1[n(k)] + I_2 \mathcal{F}_2[n(k)] \Big]$$

$$\big(\dot{n}(k)\big)_{\text{qu}} = \zeta \tilde{J}^2 \Big[\Big((3 - 2\gamma - \ln(\tilde{J}/4)) I_2 - I_{2I} \Big) \mathcal{F}_3[n(k)] + I_2 \mathcal{F}_4[n(k)] \Big]$$

I_2, I_{2I}, γ are constants of order 1.

[Functionals and constants are defined in Appendix 2]

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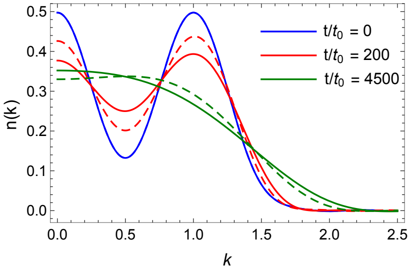
Rate of thermalization

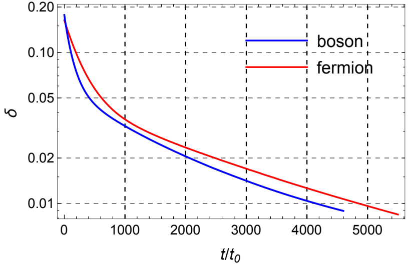
Thermalization rate $\sim m\omega^2 a_s^2 \, \tilde{J}^2 (a - b \ln \tilde{J})$, where a and b are set by $n(k)$ and grow with density.

[Functionals and constants are defined in Appendix 2]

Sample plot for $\tilde{J} = 0.001$ (with $t_0 = \pi\hbar/(2m\omega^2 a_s^2)$)



Evolution of the momentum distribution.
Solid $\to$ boson, Dashed $\to$ fermion



Mismatch between $n(k)$ and its thermal fit.

$$\implies t_{\text{th}} \sim 5000 t_0 \text{ for } \tilde{J} = 0.001$$

Equlibration time in the experiments

Rice (^{6}Li) :

- $\omega = 4\pi \times 10^5$ Hz, $a_s =$ -484 nm $\implies t_0 = 45$ ns.
- $a = 532$ nm, $V_0/E_R \approx 12$ $\implies \tilde{J} \approx 0.15$
- density $\approx 10^7 \text{cm}^{-1} \approx 5/a$

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Measurement timescale ~ 5 ms

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- $\omega = 2\pi \times 67$ kHz, $a_s = 5.3$ nm
  $\implies t_0 = 0.23$ ms.
- $a = 386.5$ nm, $V_0/E_R \approx 76$
  $\implies \tilde{J} \approx 1.5 \times 10^{-5}$
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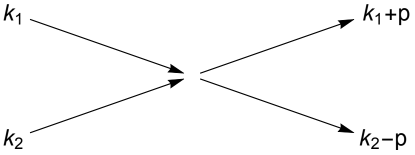
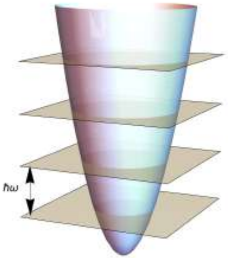
Explains the apparently conflicting observations

Dynamics with populations in higher energy bands

Lattice turn on is fast compared to τ_{coll}

Harmonic confinement in x-y plane





$$\frac{(k_1+p)^2}{2} + \frac{(k_2-p)^2}{2} = \frac{k_1^2}{2} + \frac{k_2^2}{2} + \Delta\varepsilon_\perp$$

$$\Delta\varepsilon_\perp = (\text{Integer})\ \hbar\omega$$

The initial z-momentum distribution is narrow compared to $1/d_{\text{osc}}$.

Results – three different timescales

- Degenerate states within a band : $t_{\text{th}} \sim$ 2-3 τ_{coll}
- Different energy bands : $t_{\text{th}} \sim 30\ \tau_{\text{coll}}$
- z-momentum distribution $N(k)$: $t_{\text{th}} \sim 200\ \tau_{\text{coll}}$

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Rate equation for occupations in the M-th energy band :

$$\dot{n}_M(k) = \frac{1}{M+1} \sum_{M_1, M_2, M_3 = 0}^{\infty} F(M, M_1, M_2, M_3) \int' \frac{dp}{q} \times$$

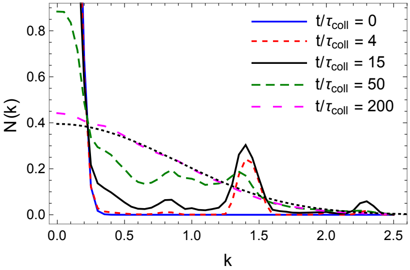
$$\Big[n_{M_1}(p+q) n_{M_2}(p-q) \left(1 + \zeta n_M(k)\right) \left(1 + \zeta n_{M_3}(2p-k)\right)$$

$$- \left(1 + \zeta n_{M_1}(p+q)\right) \left(1 + \zeta n_{M_2}(p-q)\right) n_M(k) n_{M_3}(2p-k) \Big]$$

$$q = \sqrt{(p-k)^2 + M + M_3 - M_1 - M_2},$$

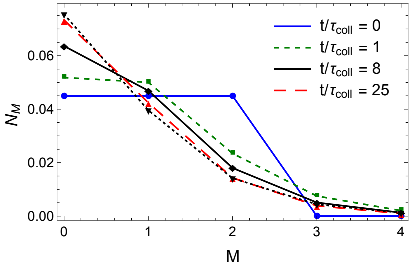
[F is defined in Appendix 3]

Sample plots



$N(k)$: total population in state k

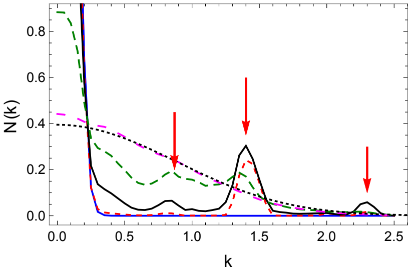
$$t_{\text{th}} \sim 200\ \tau_{\text{coll}}$$



N_M : total population in energy level M

$$t_{\text{th}} \sim 30\ \tau_{\text{coll}}$$

Peaks in $N(k)$



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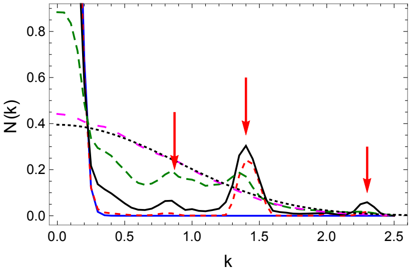
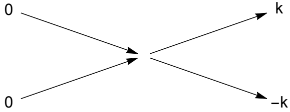


Initially, all particles have $k \approx 0$



$$2(k^2/2) = \Delta M = 0, \pm 2, \pm 4, \ldots$$

$$\Delta M = 2 \;\text{ gives }\; k = \sqrt{2} \approx 1.4$$

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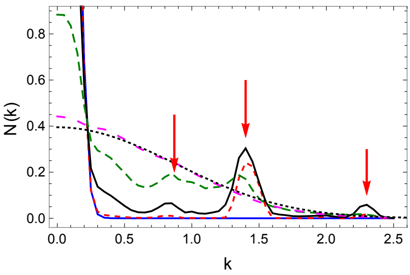

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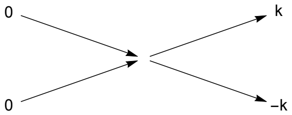

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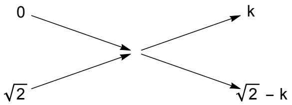
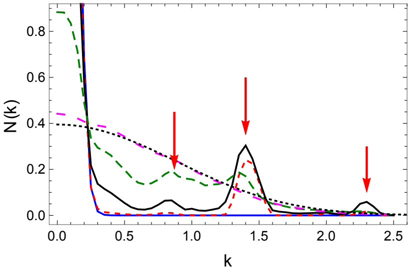


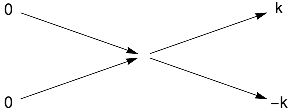
$$\Delta M = 2 \;\; \text{gives} \;\; k \approx 0.87, 2.3$$

Peaks in $N(k)$



Can be probed in future experiments

Initially, all particles have $k \approx 0$



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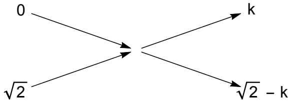

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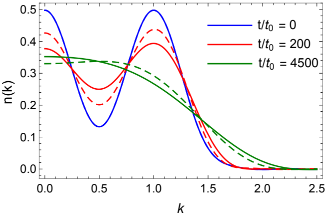
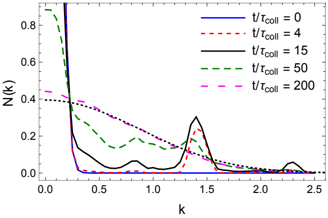
Summary

– Particles do thermalize after sufficiently many collisions if

- they are in the lowest band, and $\tilde{J} \neq 0$: $n(k)$ changes smoothly to a thermal profile
- they occupy more than one energy levels : $n(k)$ exhibits isolated peaks which later merge and form a thermal profile





– Thermalization rate in a single band $\propto m\omega^2 a_s^2 \tilde{J}^2 (a - b \ln \tilde{J}) + \mathcal{O}(\tilde{J}^4 \ln \tilde{J})$

Appendix 1 [go back to slide]

$$f(\xi) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} du \; e^{i\xi u} \left({}_2F_3\left(\frac{1}{2}, \frac{1}{2}; 1, 1, 1; -4u^2 \right) \right)^2$$

${}_2F_3$ denotes a Hypergeometric function

$f(\xi)$ is peaked about $\xi = 0$ and vanishes for $|\xi| > 8$

$$I_2 \equiv \int_0^{\infty} d\xi \; \xi^2 f(\xi) \approx 2$$

$$I_{2l} \equiv \int_0^{\infty} d\xi \; \xi^2 \ln \xi \; f(\xi) \approx 2.18$$

$\gamma \equiv$ Euler-Mascheroni constant ≈ 0.577

Appendix 2 [go back to slide]

$$\mathcal{F}_1[n(k)] \equiv (n''(k))^2 - n(k)n^{(4)}(k)$$

$$\mathcal{F}_2[n(k)] \equiv n(k)\int_0^\infty dp\ln p\,\left(n^{(5)}(k+p) - n^{(5)}(k-p)\right) - n''(k)\int_0^\infty dp\ln p\,\left(n^{(3)}(k+p) - n^{(3)}(k-p)\right)$$

$$\mathcal{F}_3[n(k)] \equiv -\frac{n(k)}{3}\left[6(n''(k))^2 + 16n'(k)n^{(3)}(k) + 5n(k)n^{(4)}(k)\right]$$

$$\mathcal{F}_4[n(k)] \equiv 2\,(\ln 2 - \gamma)\,\partial_k^2\Big((n(k))^2 n^{(3)}(k) - 2(n'(k))^3 - 2n(k)n'(k)n''(k)\Big) + \partial_k^2\Big((n(k))^2\int_0^\infty dp\ln p\,\left(n^{(4)}(k+p) - n^{(4)}(k-p)\right) - n'(k)\,\partial_k^3\int_0^\infty dp\ln p\,\left((n(k+p))^2 - (n(k-p))^2\right)\Big)$$

where $n^{(i)}(k) \equiv \partial_k^i n(k)$

$$F(M, M_1, M_2, M_3)$$

$$\equiv \sum_{n=0}^{M} \sum_{n_1=0}^{M_1} \sum_{n_2=0}^{M_2} \sum_{n_3=0}^{M_3} \mathcal{C}(n, n_1, n_2, n_3)\, \mathcal{C}(M-n, M_1-n_1, M_2-n_2, M_3-n_3)$$

where

$$\mathcal{C}(n, n_1, n_2, n_3) = \begin{cases} 0 \text{ if } n+n_1+n_2+n_3 \text{ is odd} \\ \frac{1}{n!n_1!n_2!n_3!} \left(\frac{\Gamma(\frac{n+n_3+n_1-n_2+1}{2})\Gamma(\frac{n-n_3+n_1+n_2+1}{2})}{\Gamma(\frac{n-n_3+n_1-n_2+1}{2})} \right)^2 \times \\ \left({}_3F_2 \left(\begin{matrix} -n, & -n_1, & \frac{-n+n_3-n_1+n_2+1}{2}; \\ \frac{-n-n_3-n_1+n_2+1}{2}, & & \frac{-n+n_3-n_1-n_2+1}{2}; \end{matrix} \; 1 \right) \right)^2 \\ \text{if } n+n_1+n_2+n_3 \text{ is even} \end{cases}$$