

1. Figure 1 shows a **conducting** sphere which has a rectangular hole in it. A point charge  $q_1 = +2\mu C$  is placed inside the hole and another point charge  $q_2 = +5\mu C$  is placed outside the sphere. The conducting *sphere itself* has zero total charge.

(a) What is the net charge on the inner surface of the sphere (i.e., on the sides of the hole)? (4 points)

*Hint: Apply Gauss' law to a closed surface that encloses the hole.*

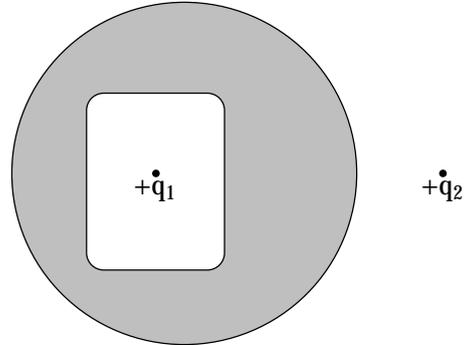


Figure 1

(b) What is the net charge on the outer surface of the sphere? (3 points)

2. (a) A metal rod is kept in contact with two neutral **conducting** spheres A and B, and a positively charged body is brought close to sphere B, as shown in Figure 2.

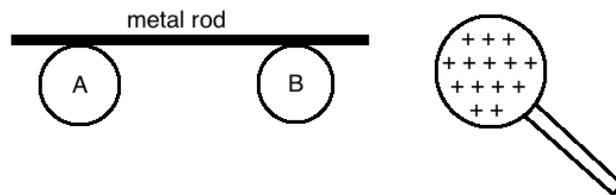


Figure 2

Then, keeping the charged body in the same position, the metal rod is taken away.

Which is true about the **final** charges in the spheres A and B? (3 points)

- i) Both spheres are negatively charged.
- ii) Each sphere has  $-ve$  charges to its right and  $+ve$  charges to its left. The total charge on each is zero.
- iii) Sphere A is positively charged and sphere B is negatively charged. We have an electric dipole.
- iv) Sphere B is positively charged and sphere A is negatively charged. We have an electric dipole.

Please turn over

(b) If the two spheres A and B were non-conducting, which would be true about the final charges in them? (3 points)

- i) Both spheres are negatively charged.
- ii) Each sphere has  $-ve$  charges to its right and  $+ve$  charges to its left. The total charge on each is zero.
- iii) Sphere A is positively charged and sphere B is negatively charged. We have an electric dipole.
- iv) Sphere B is positively charged and sphere A is negatively charged. We have an electric dipole.

3. *Lightning on airplane!*

Most aircraft bodies are made out of Aluminium. It's estimated that lightning strikes each plane in the U.S. commercial fleet about once a year.



Figure 3: lightning strikes an airplane!

How big an issue is lightning to passengers? Choose your answer: (3 points)

- i) It cannot penetrate inside the aircraft since its body is a conductor.
- ii) It cannot penetrate inside the aircraft since the aircraft skin is very thick.
- iii) It can penetrate inside the aircraft and give you a shock.
- iv) It is sure to cause a plane crash!

4. Consider the circuit shown in Figure 4.  $Q$  denotes the charge on the capacitor  $C$ , and  $i_1$  and  $i_2$  denote the currents in the resistors  $R_1$  and  $R_2$  respectively. For  $t < 0$ , the double-throw switch  $S$  is kept open at position 1, and the capacitor plates have no charge. At  $t = 0$ , the switch is quickly moved to position 2.

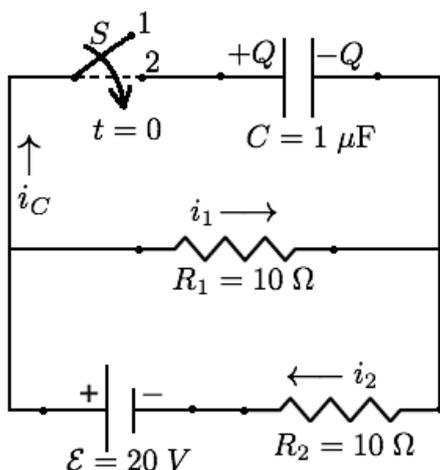


Figure 4

(a) Just after  $t = 0$ , what are the values of the following quantities? (6 points)

- i)  $Q =$
- ii)  $i_1 =$
- iii)  $i_2 =$

Please turn over

(b) After a long time, when steady state is reached, what values do the charge and currents take ? (6 points)

i)  $i_1 =$

ii)  $i_2 =$

iii)  $Q =$

*Hint: In steady state, is there any current through the capacitor ?*

5. You are given three  $2 \mu\text{F}$  capacitors. How would you connect them to get an equivalent capacitance of  $3 \mu\text{F}$  ? Draw below. (4 points)

6. A current  $i$  is driven through a rectangular sample of some material, as shown in Figure 5. Throughout the sample, there exists a uniform magnetic field  $B$  pointing into the page.  $L$  and  $R$  denote the two sides of the sample.

For parts (a) and (b), assume that the current is carried by electrons in the sample.

(a) Which side are the electrons deflected toward by the magnetic field ?  $L$  or  $R$  ? (3 points)

(b) In steady state, which of the two sides,  $L$  or  $R$ , is at a higher electric potential ? (3 points)

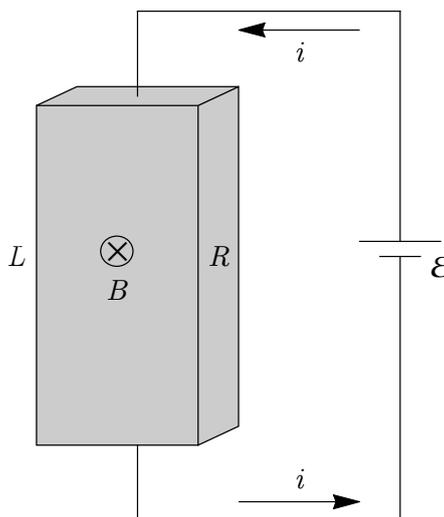


Figure 5

Please turn over

(c) If the same current in the sample were carried by positive ions in the sample, instead of electrons, which side would have a higher electric potential ? (3 points)

(d) If the current is increased by increasing the battery emf  $\mathcal{E}$ , will the potential difference between the two sides increase or decrease in magnitude ? (3 points)

7. (4 points) Consider the situation shown in Figure 6. A clockwise current flows along a square loop which is placed close to a long straight wire carrying a current in the upward direction.

Under the influence of the magnetic field of the long wire, the loop will

- i) move towards the wire.
- ii) move away from the wire.
- iii) remain stationary.
- iv) rotate about the wire.

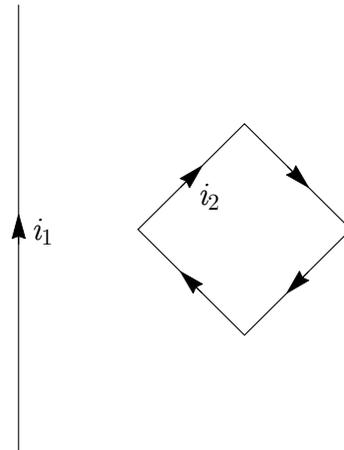


Figure 6

Please turn over

8. Figure 7 shows an Amperian loop  $C$  and a number of currents. If the circulation of the net magnetic field (counterclockwise along  $C$ ) is

$$\oint \vec{B} \cdot d\vec{l} = -4\pi \times 10^{-7} \text{ T}\cdot\text{m} ,$$

find the value of the current  $i_0$  shown in the diagram. (4 points)

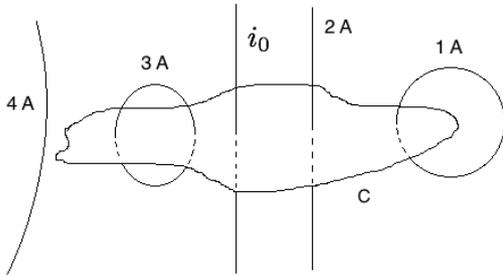


Figure 7

9. If a *non-conducting* loop is placed in a time-varying magnetic field, which of the following is true? (3 points)

- (i) Both the induced emf and the induced current are zero.
- (ii) Both the induced emf and the induced current are non-zero.
- (iii) The induced emf may be non-zero but the induced current is zero.

10. Two conducting loops are moved in the vicinity of a long straight wire carrying a current  $i$  in the downward direction, as shown in Figure 8.

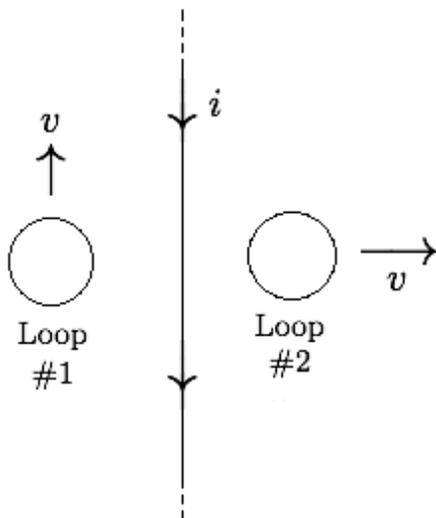


Figure 8

(a) Which is true about the induced currents in the two loops if  $v > 0$ ? (4 points)

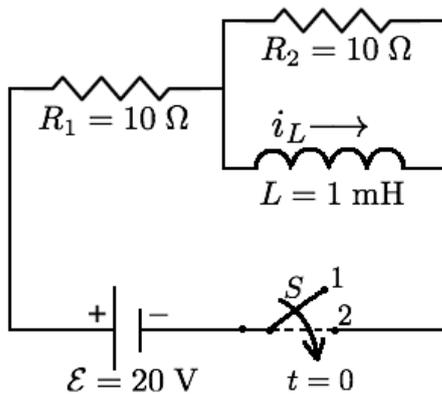
- (i) #1: clockwise, #2: counterclockwise
- (ii) #1: zero, #2: clockwise
- (iii) #1: zero, #2: counterclockwise
- (iv) None of the above

Please turn over

(b) Keeping the loops stationary ( $v = 0$ ), the direction of the current  $i$  is flipped over some time interval. Which is true about the induced currents within that time interval? (4 points)

- (i) #1: clockwise, #2: counterclockwise
- (ii) #1: counterclockwise, #2: clockwise
- (iii) #1: zero, #2: counterclockwise
- (iv) #1: zero, #2: clockwise

11. Consider the circuit shown in Figure 9.  $i_L$  denotes the current through the inductor. The switch  $S$  is closed at  $t = 0$ . Before closing the switch, there was no current in the circuit.



(a) Just after  $t = 0$ ,  $i_L = ?$  (2 points)

(b) After a long time, steady-state is reached, i.e., currents are not changing any more. Then,  $i_L = ?$  (3 points)

Figure 9

12. You may have seen the warning “Objects in mirror are closer than they appear” on side-view mirrors in cars. This means that for such mirrors the image is always smaller than the object.

(a) Are these mirrors convex, concave, or straight? (2 points)



Figure 10  
Please turn over

(b) Suppose such a mirror has a focal length of magnitude 1 m. If an object is 1 m behind the mirror, by what factor will its height be reduced in the image ? (3 points)

**13.** A coin is glued to the bottom of a container. Suppose you are looking straight at the corner point  $P$  as shown in Figure 11. Without any liquid in the container, the coin is not visible. You start pouring liquid until its height becomes  $h$ , when you can see the coin.

(a) In Figure 11, draw the light ray that starts from the coin and reaches your eye. (2 points)

(b) If the liquid has a refractive index  $n = \sqrt{2}$ , what is the incident angle in the liquid for this ray ? (3 points)

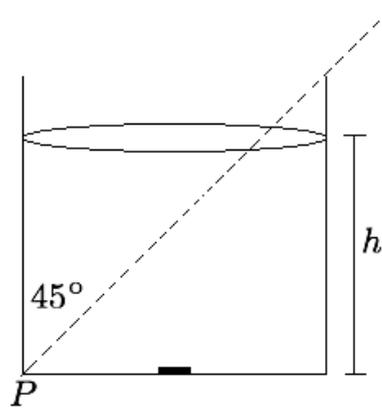


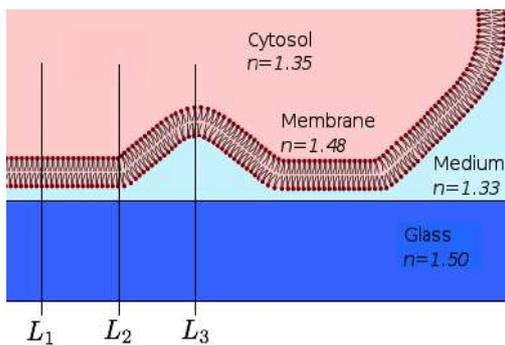
Figure 11

(c) Oil has a higher refractive index than water. For which of these two liquids will  $h$  be larger ? (2 points)

*Hint: light bends more for higher refractive index. You don't need any calculation!*

**14.** Interference Reflection Microscopy is a technique often used to study cell-substrate adhesion patterns. Figure 12 shows how this works. A cell is separated from a thick glass substrate by a thin film of culture medium. Depending on how the cell membrane adheres to the glass at different places, the film thickness varies. One can track this variation by shining light and seeing how strongly it is reflected from different parts of the film.

Assume that green light of wavelength (in vacuum) 532 nm is incident normally on the film from the glass.



(a) Suppose the film thickness is negligible along line  $L_1$  (or  $L_2$ ). Shall we get strong reflection or no reflection at these locations ? (2 points)

Figure 12

(b) Suppose the light is strongly reflected along line  $L_3$ . What is the minimum possible film thickness at this location ? (3 points)

**15.** Figure 13 shows an infrared image of the Horsehead Nebula taken by the Hubble Space Telescope. Hubble uses a parabolic mirror of diameter 2.44 m. It took the image using light of wavelength 200 nm, and could barely resolve two points on the nebula which are separated by  $2 \times 10^{12}$  m (50000 times the earth's circumference)! Find out how far the Horsehead Nebula is from earth. (4 points)



Figure 13

Please turn over

16. A source of light  $S$  is placed at a height  $h$  above a mirror, as shown in Figure 14. Light can reach a point on the screen to the right either directly or after getting reflected. The direct and the reflected ray interfere to produce fringes.

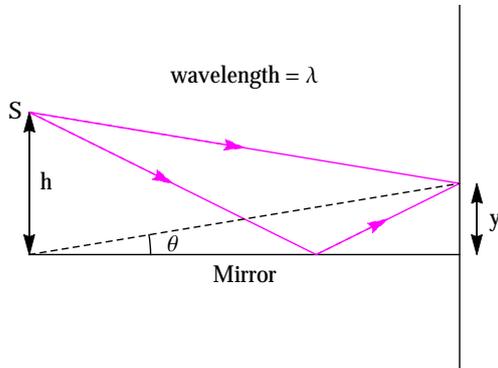


Figure 14

(a) Shall we see a bright fringe or a dark fringe near  $y = 0$  ? (2 points)

(b) One can show that the reflected ray travels an extra distance of  $2h \sin \theta$ . What is the condition for getting a dark fringe at angle  $\theta$  ? (4 points)

(c) Take  $\lambda = 0.5 \mu\text{m}$  and  $h = 50 \mu\text{m}$ . Find out the total number of dark fringes on the screen. (3 points)

17. If oil is used instead of air as the medium between the specimen and the lens in a microscope, will its resolution increase, decrease, or stay the same ? (2 points)

Please turn over

1. You are told that an infinite sheet of charge produces a uniform electric field of magnitude  $\sigma/(2\epsilon_0)$ , where  $\sigma$  is the charge per unit area. The field points away from the sheet for positive charge and towards it for negative charge.

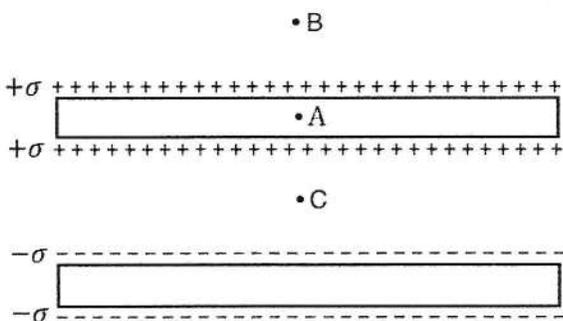


Figure shows two parallel plates with charges on their four surfaces.  $+\sigma$  or  $-\sigma$  gives the charge per unit area. Each surface resembles an infinite sheet of charge.

(a) Find the net electric field at Point A (inside the upper plate) produced by the four charge sheets (Don't forget to include directions when you add the fields).

(b) Find the net electric fields at Point B and Point C.

(c) Let  $E_A$ ,  $E_B$ , and  $E_C$  denote the magnitudes of the net electric fields at A, B, and C. Which of the following is true?

- i)  $E_A < E_B < E_C$     ii)  $E_B < E_A < E_C$     iii)  $E_A < E_B = E_C$

(d) If the plates are made of conductors, can we get such a charge distribution (as given in figure) in electrostatic equilibrium? Why or why not?

1. "Sirius" is the brightest star in the night sky. Its distance from the earth is  $5 \times 10^5$  times the earth-sun distance. Suppose a measurement of the light intensity ( $I$ ) at the earth shows that  $I_{\text{Sirius}} = 10^{-10} I_{\text{sun}}$ . Assuming the inverse square law for intensity, calculate the ratio of the powers emitted, i.e.,  $P_{\text{Sirius}}/P_{\text{sun}}$ .

2. A coin is glued to the bottom of a non-transparent container. Suppose you are looking straight at the point  $P$  as shown in Fig. 1. Without any liquid in the container, the coin is not visible. You start pouring liquid until its height becomes  $h$ , when you can see the coin.

(a) In Fig. 1, draw the light ray that starts from the center of the coin, and reaches your eye.

(b) If the liquid used has a refractive index  $n = \sqrt{2}$ , what is the incident angle in the liquid for this ray?

(c) Vegetable oil has a higher refractive index than water. For which of the two liquids will  $h$  be larger? (You don't need any calculation.)

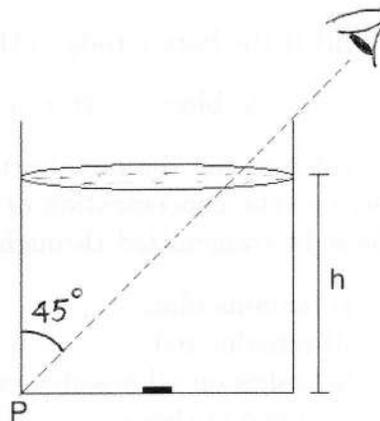


Figure 1

3. If light is passed through a sugar solution, it rotates the direction of polarization. The amount of rotation  $\Delta\theta$  depends on wavelength.

$$\text{For red light, } \Delta\theta_{\text{red}} \text{ (in degrees)} = 6cl,$$

$$\text{For blue light, } \Delta\theta_{\text{blue}} \text{ (in degrees)} = 12cl,$$

where  $c$  is the concentration of the solution in g/ml, and  $l$  is the length of the solution in cm.

Suppose you shine linearly polarized purple light (an equal mixture of red and blue) through a 1 g/ml sugar solution.

Please turn over

(a) What minimum length of solution will make the outgoing red and blue light polarized perpendicular to each other?

(b) For this length and concentration, suppose the outgoing red and blue light are polarized along  $x$  and  $y$  respectively. You hold a metal grill in the path of this light and see the transmitted light from the other side.

i) If the bars / rods in the grill are parallel to  $x$ , you will see

A. blue    B. red    C. purple    D. none of these

ii) If the bars / rods in the grill are parallel to  $y$ , you will see

A. blue    B. red    C. purple    D. none of these

iii) If the bars / rods in the grill are equally inclined to  $x$  and  $y$ , you will see

A. blue    B. red    C. purple    D. none of these

(c) Keeping the metal grill fixed with its bars parallel to  $x$ , you continuously increase the concentration of the solution by adding more sugar. As you do this, the light transmitted through the grill

i) remains blue.

ii) remains red.

iii) takes on all possible colors obtained by mixing red and blue.

iv) none of these.

Please turn over

1. A point object  $O$  is placed in front of a thin lens whose upper half has a refractive index  $p$  and lower half has a refractive index  $q$ , as shown in Fig. 1. If instead the *entire* lens had index  $p$  (or  $q$ ), an image would form at  $P$  (or  $Q$ ).

For the lens shown in Fig. 1, which of the following image(s) shall we see?

- i) A single image at a point midway between  $P$  and  $Q$ .
- ii) No image at all.
- iii) A blurred image between  $P$  and  $Q$ .
- iv) Two images, one at  $P$ , and the other at  $Q$ .

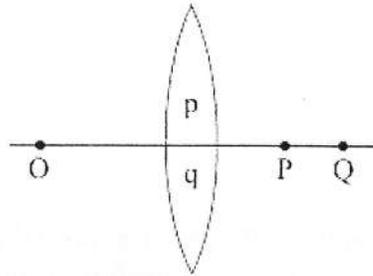


Figure 1

2. Figure 2 shows the path of a light ray in a primary rainbow, which undergoes two refractions and one reflection in a water drop. It is known that the incident angle for the reflection is about  $40^\circ$ . Some part of the incident light might also be refracted into air at point  $P$ , in which case, they are not shown.

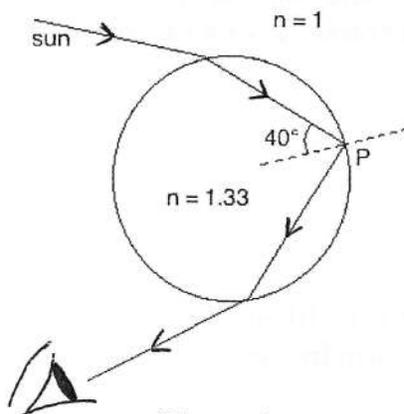


Figure 2

(a) Is it total internal reflection or normal reflection at point  $P$ ?

(b) Find out the Brewster's angle for this reflection. How does it compare with the actual angle of incidence?

(c) If you look at the rainbow wearing polaroid sunglasses (vertical transmission axis), you would see

- i) as bright a rainbow as with the naked eye.
- ii) half as bright a rainbow as with the naked eye, as it is unpolarized.
- iii) very dim rainbow (if at all) since it is strongly polarized.
- iv) none of these.

Please turn over

## FUN PROBLEM

*Find out the size of an atom and its energy levels yourself!*

According to de Broglie, in the  $n$ -th energy state of the Hydrogen atom, the electron moves in a circular orbit such that the orbit fits  $n$  number of de Broglie wavelengths. Figure shows two such examples for  $n = 5$  and  $n = 6$ , and the only smiling photo of de Broglie I could find!

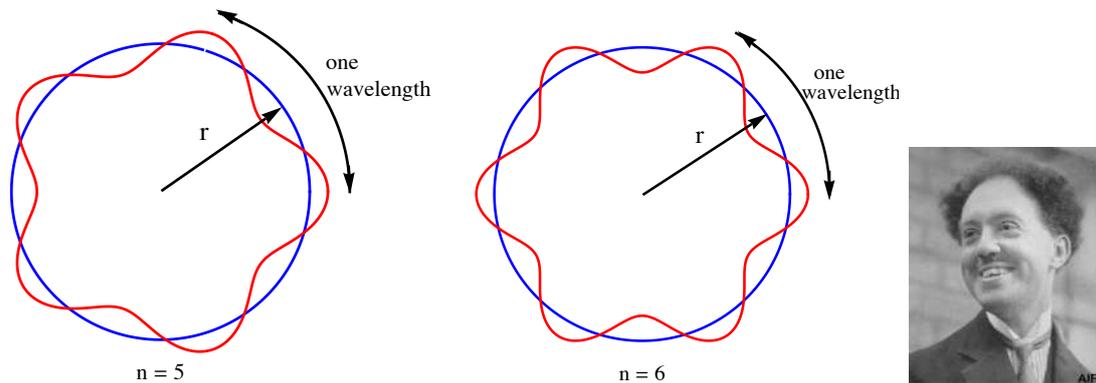


Figure 1: Electron orbits for the  $n = 5$  and the  $n = 6$  states, and de Broglie.

(a) Find the wavelength  $\lambda$  in the  $n$ -th energy state in terms of the radius  $r$ .  
(Circumference of a circle =  $2\pi r$ !)

(b) What is the momentum  $p$  of the electron for the wavelength you found?  
**Don't** substitute numerical value of  $h$ . ( $p = h/\lambda$ , remember?)

(c) Using  $K = p^2/(2m)$ , find the kinetic energy of the electron in the  $n$ -th energy state. **Don't** substitute numerical values of  $h$  and  $m$ !

(d) In the first part of this course, you learned that the potential energy of two point charges  $q_1$  and  $q_2$  at a distance  $r$  is  $q_1 q_2 / (4\pi\epsilon_0 r)$ . Write down the expression for the potential energy  $U$  of the electron (charge  $-e$ ) and the nucleus (charge  $e$ ) in terms of the radius  $r$ . **Don't** substitute numerical values of  $e$  and  $\epsilon_0$ !

(e) Last semester you probably learned about the Virial theorem (remember?) which says that for a circular orbit  $K = -U/2$ . Let's apply this theorem to what you found in (c) and (d), and find out the radius  $r$  in terms of  $h$ ,  $m$ ,  $e$ ,  $\epsilon_0$ , and  $n$ . **Don't** substitute the numerical values yet.

(f) Now that you've got the radius, find out the total energy  $E = K + U = U/2$  by substituting for  $r$  in part (d) and dividing by 2.

(g) Using the following values of the universal constants, find out the radius and energy of the ground state ( $n = 1$ )! (If you need,  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ .)

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-2} \text{ m}^{-2}$$

$$e = 1.60 \times 10^{-19} \text{ C}$$

$$\pi = 3.14159265358979323846264338327950288419716939937510 \dots \text{ ☺}$$