Creating Laughlin states and braiding anyons in an optical cavity

Shovan Dutta and Erich J. Mueller

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853

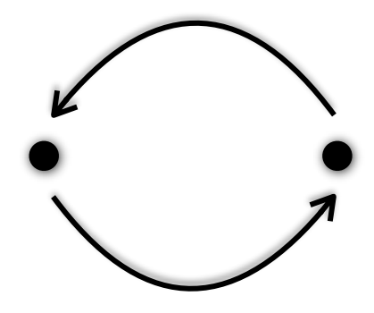
Phys. Rev. A 97, 033825 (2018)

ARO-MURI on Non-Equilibrium Dynamics
NSF PHY-1508300

I. Motivation

Anyons:

- Fractional exchange statistics — new physics



Bosons: $+1$	Fermions: -1
Anyons: $e^{i\phi_s}$	Non-Abelian: U

- Potential for topological quantum computation [1]
- Anyons exist in fractional quantum Hall effect [2] (2D electrons in magnetic field) — hard to observe

Q: How to create anyons and measure statistics?

II. Goal

- Create fractional quantum Hall states with light (polaritons)

  Simplest analog: $\nu = 1/2$ Laughlin state:

$$\Phi_N(z_1,\ldots,z_N) = \prod_{j<k}(z_j - z_k)^2\, e^{-\sum_i |z_i|^2/2}$$

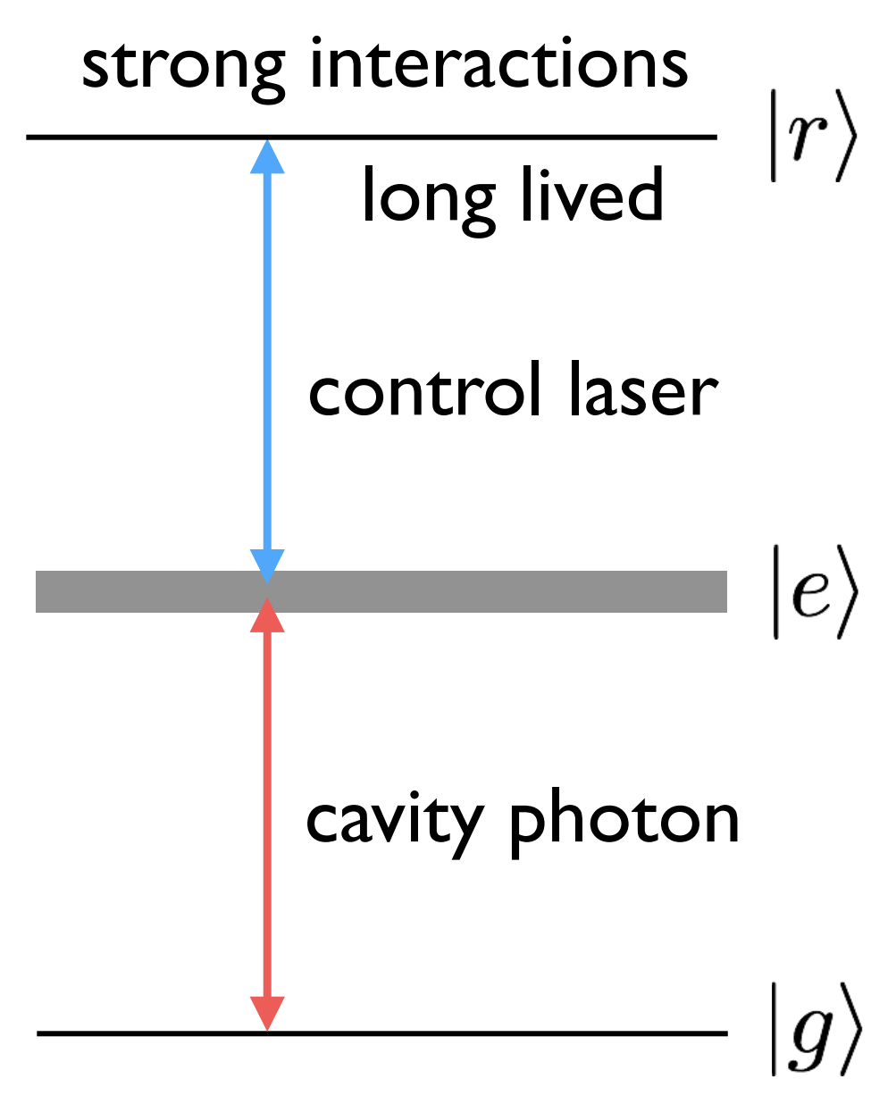
  $z_j \equiv (x_j + iy_j)/l$: location of j^{th} particle, l: magnetic length

- Create anyonic quasihole excitations at $\pm z_0$:

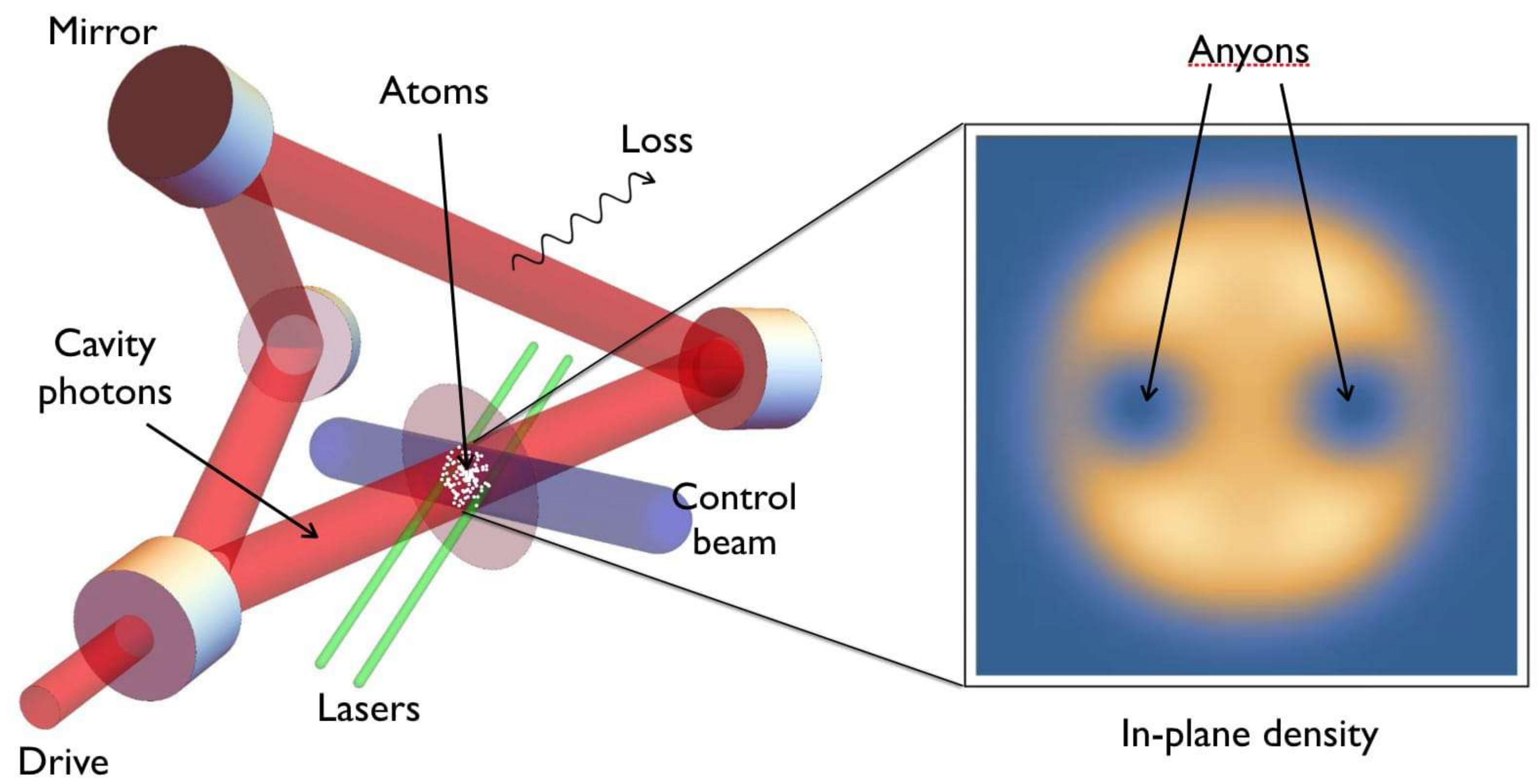
$$\Phi_N^{\circ\circ}(\{z_i\}) = \prod_j (z_j - z_0)(z_j + z_0)\,\Phi_N(\{z_i\})$$

- Exchange quasiholes and measure statistics

III. Envisioned experiment



Atom-photon coupling (Rydberg EIT)



In-plane density

- Near-degenerate cavity — longitudinal mode number fixed ⇒ effective 2D dynamics in a transverse plane
- Non-planar geometry rotates transverse light field ⇒ effective magnetic field [3]
- Concave mirrors confine light ⇒ transverse harmonic trap
- Atom-light coupling mediates photon-photon interactions ⇒ long-lived strongly interacting Rydberg polaritons [4]
- Extra lasers yield localized potentials for binding holes

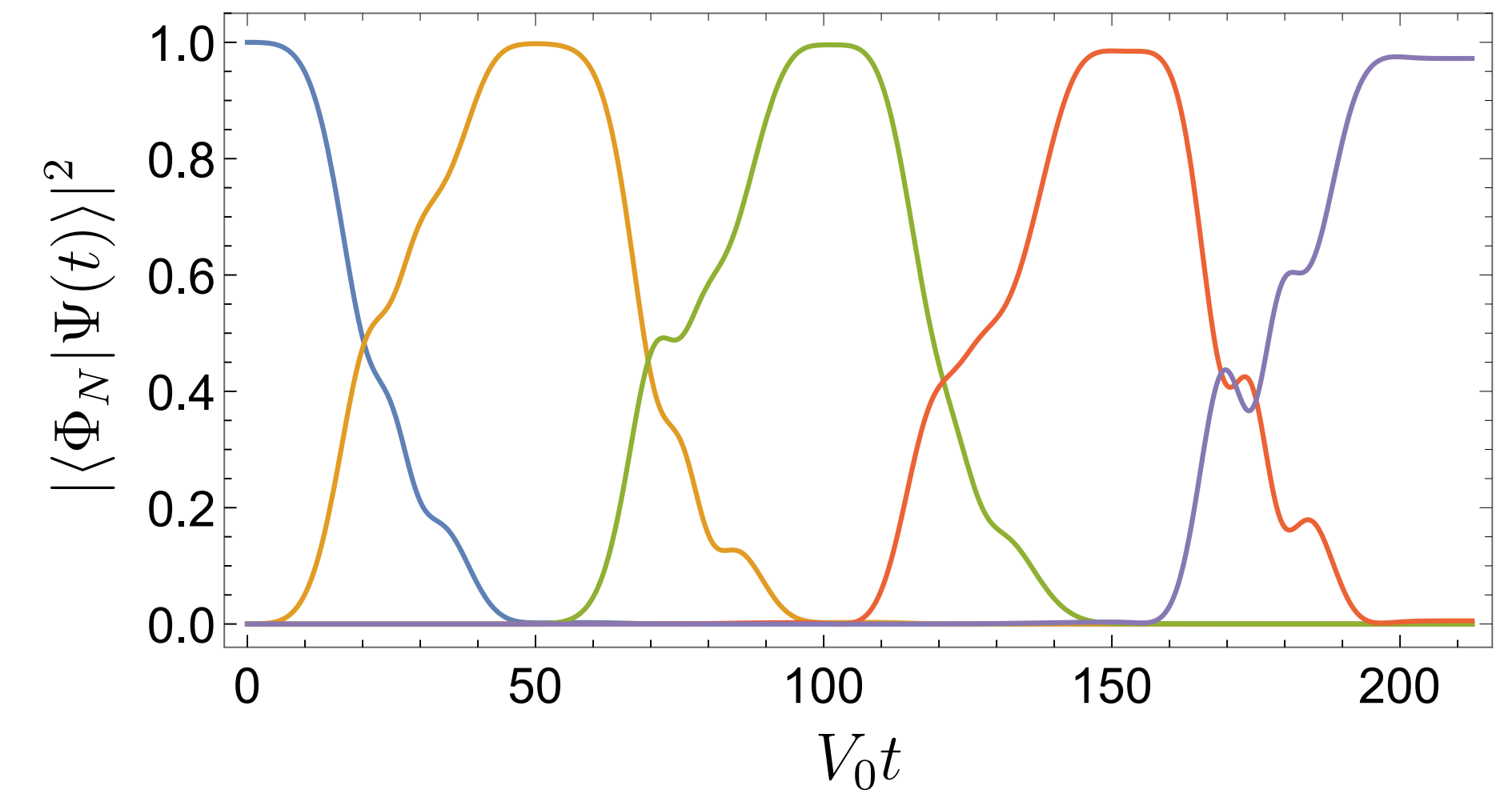
IV. Model

$$\hat{H} = \int d^2r\,\hat{\psi}^\dagger\Big[\underbrace{\frac{(-i\vec{\nabla} - M\omega_B r\hat{\varphi})^2}{2M}}_{\text{kinetic}} + \underbrace{\frac{1}{2}M\omega_T^2 r^2}_{\text{trap}}\Big]\hat{\psi} + \underbrace{\frac{\pi V_0}{2M\omega_B}\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}}_{\text{interaction}} + \underbrace{F(\vec{r},t)\,\hat{\psi}^\dagger + F^*(\vec{r},t)\,\hat{\psi}}_{\text{drive}} + \underbrace{U(\vec{r},t)\,\hat{\psi}^\dagger\hat{\psi}}_{\text{potential}}$$

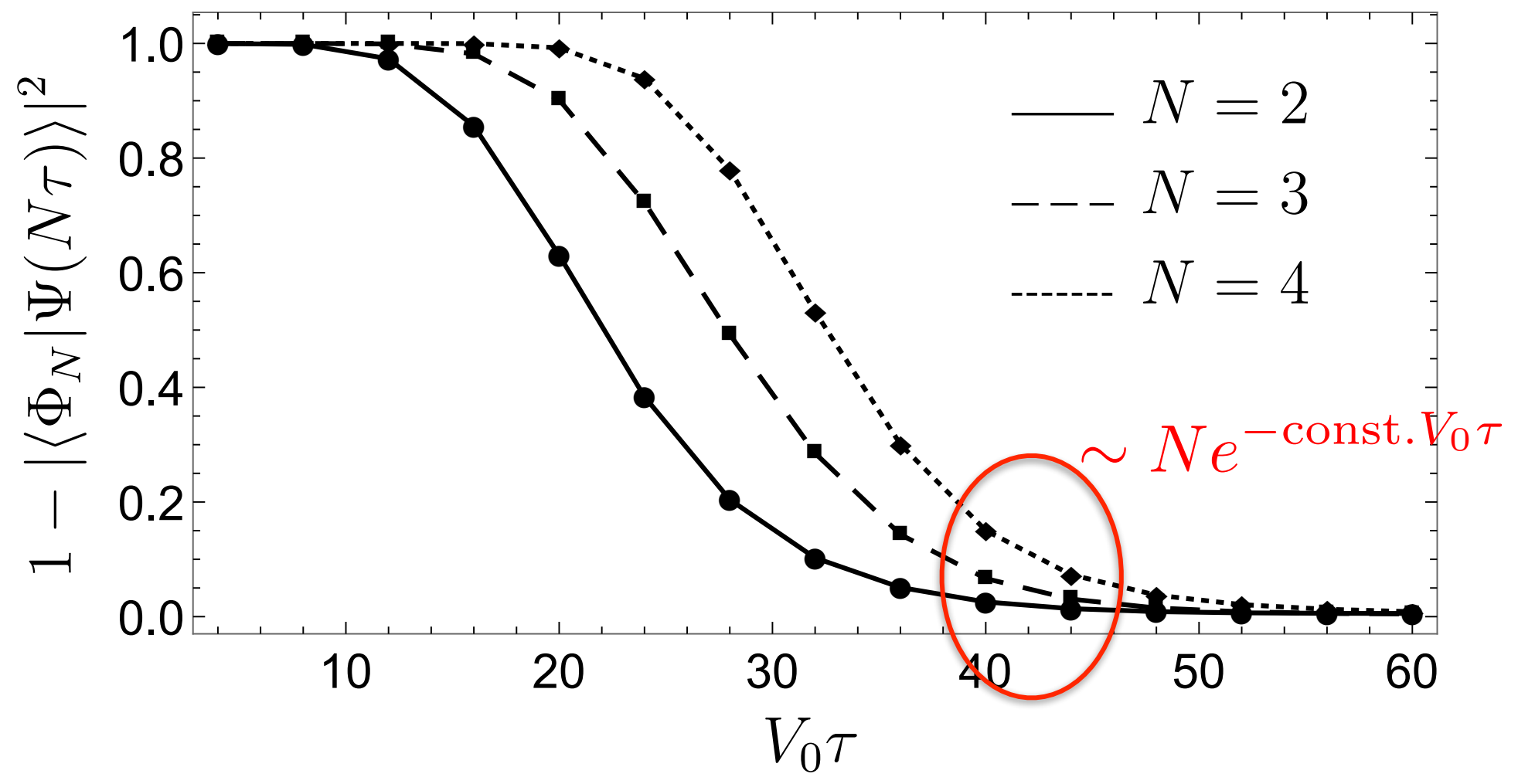
- Energy scales: (1) Landau level splitting $2\omega_B$, (2) trap freq. ω_T, (3) interaction energy V_0, (4) polariton decay rate γ
- Typically, $\omega_T, V_0, \gamma \ll \omega_B \Rightarrow$ dynamics confined to lowest Landau level, spanned by states $\phi_m(z) = z^m e^{-|z|^2/2}$
- Laughlin state $|\Phi_N\rangle$ is the N-particle ground state, with total angular momentum $L_N = N(N-1)$

V. Creating Laughlin state and quasiholes

- Rapid adiabatic passage: inject photons one-by-one such that $|\Phi_0\rangle \to |\Phi_1\rangle \to \cdots \to |\Phi_N\rangle$
  1. Pump photons with angular momentum $m = L_{n+1} - L_n = 2n$ (Laguerre-Gauss laser beams)
  2. Sweep drive frequency thru resonance to induce transition from $|\Phi_n\rangle$ to $|\Phi_{n+1}\rangle$



Overlap with each N-particle Laughlin state as a function of time, for sweep duration $\tau = 50/V_0$. Each successive plateau corresponds to increasing N by 1.



Cumulative error in state preparation as a function of sweep duration τ for different final particle numbers N. It falls off exponentially for large τ.

- Adiabatically insert strongly focused lasers to bind quasiholes and drag them around one another (A small but finite trap frequency suppresses edge excitations)
- Sweeps must be slow enough to prevent excitations ($\tau \gg 1/V_0$) and fast enough to prevent loss ($\tau \ll 1/\gamma$) $\implies V_0/\gamma \gg 10N^2 \ln N$ (current experiments have $V_0/\gamma \sim 50$)

VI. Measuring anyonic statistics

Total phase $\phi = \phi_d$ (dynamical) $+\ \phi_g$ (geometric)

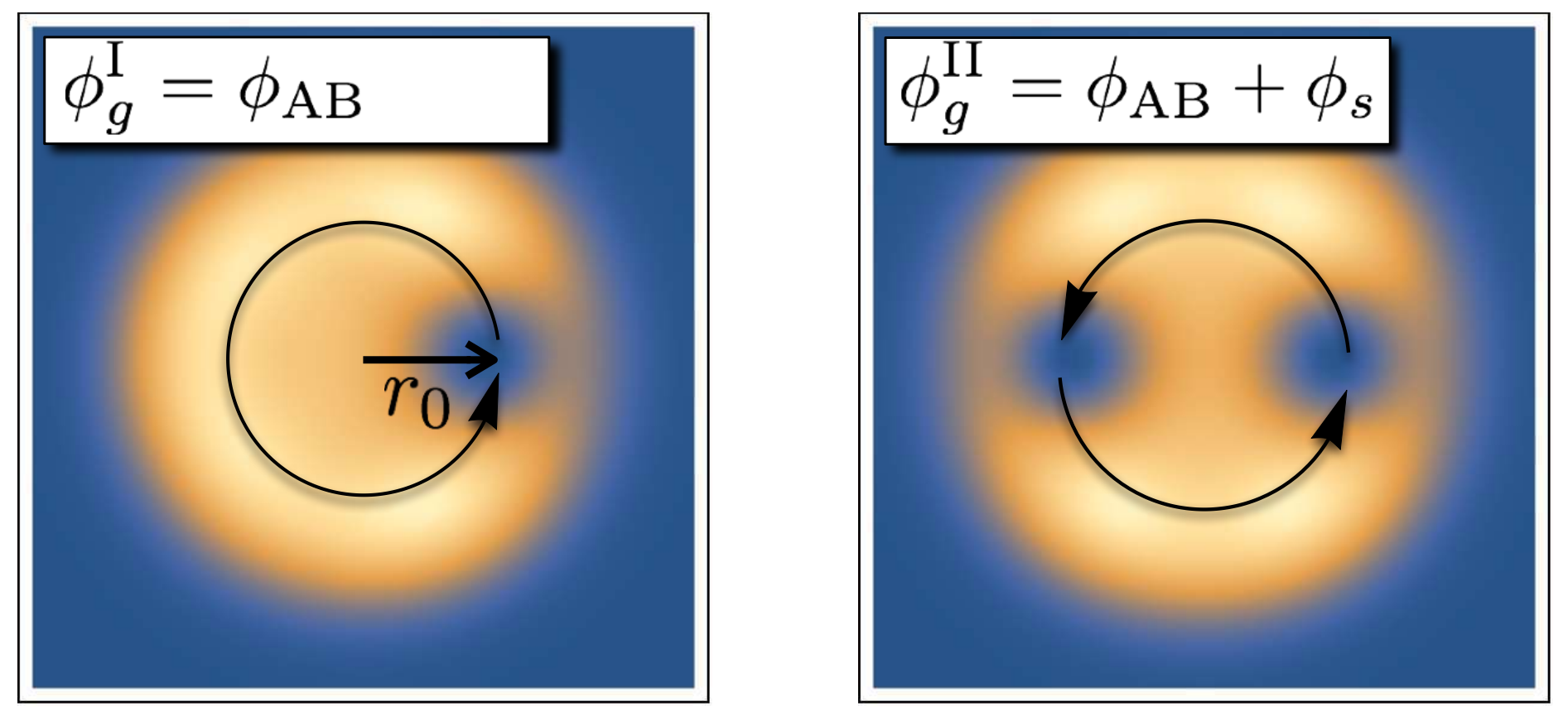
- Measuring ϕ: compare with a reference $|R\rangle$ which is unaffected by drives (e.g., a Rydberg state with large blockade radius)

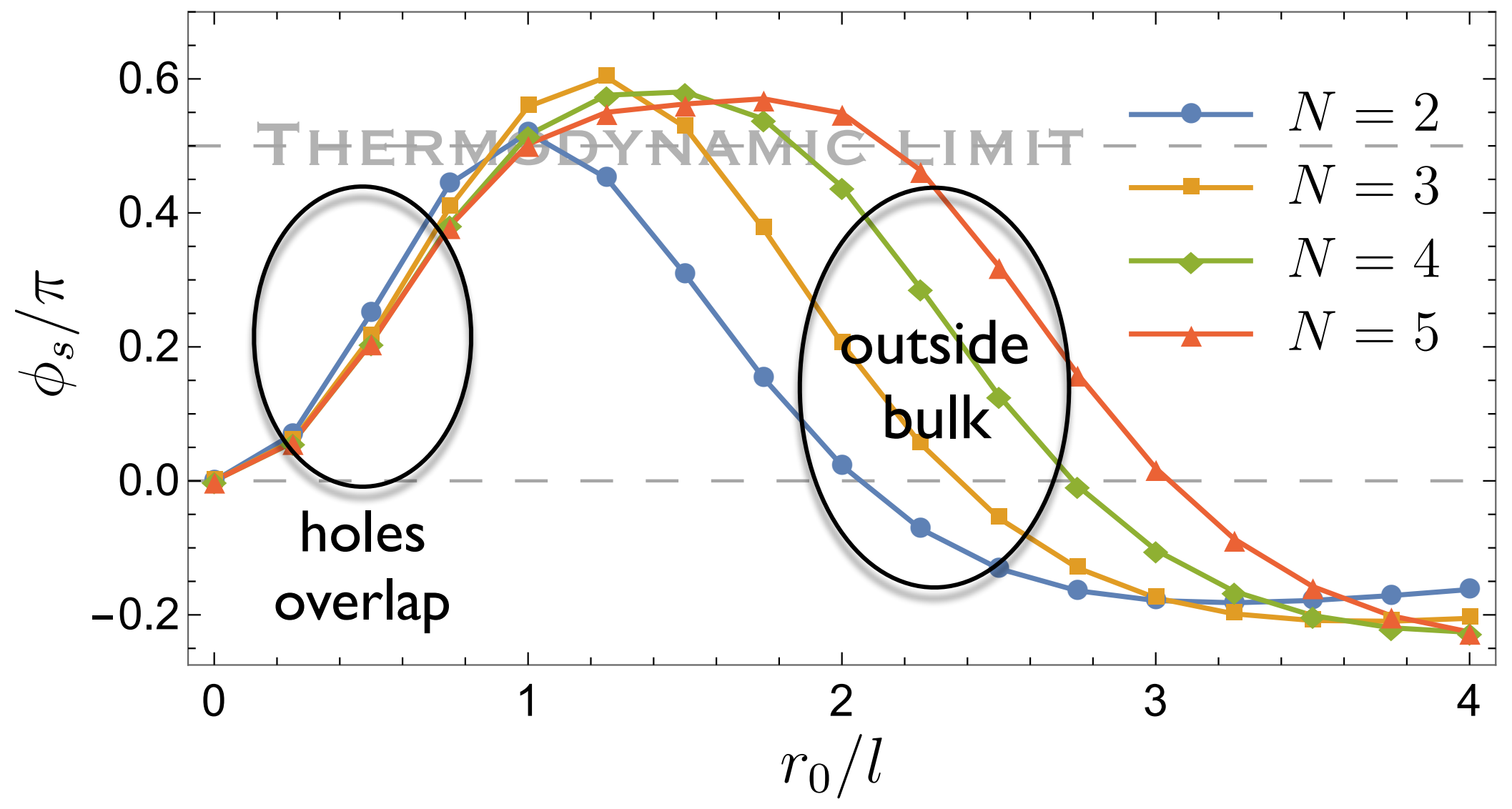
  1. Use $\pi/2$-pulse to prepare $|0\rangle + |R\rangle$
     $|0\rangle$: zero-polariton state
  2. Create Laughlin, create holes, braid holes, then repeat backward
     $|0\rangle + |R\rangle \to e^{i\phi}|0\rangle + |R\rangle$
  3. Use $\pi/2$-pulse to recombine $|0\rangle$ and $|R\rangle$
  4. Find ϕ by measuring ground-state occupation

- Repeat experiment at different rates to separate ϕ_g from ϕ_d

$\phi_g = \phi_{\text{AB}}$ (Aharonov-Bohm) $+\ \phi_s$ (statistical)

- Measuring ϕ_s: compare ϕ_g from two experiments





1. A. Y. Kitaev, Ann. Phys. **303**, 2 (2003)
2. D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984)
3. N. Schine, A. Ryou, A. Gromov, A. Sommer, J. Simon, Nature (London) **534**, 671 (2016)
4. N. Jia, N. Schine, A. Georgakopoulos, A. Ryou, L. W. Clark, A. Sommer, and J. Simon, Nat. Phys. **14**, 1 (2018)