# Protocol for creating Laughlin states of polaritons and braiding anyons


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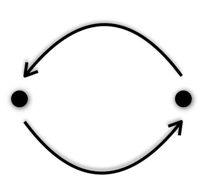
## I. Goal and Motivation

Goal:

- Create analog of fractional quantum Hall state with Rydberg polaritons
- Braid quasiholes and measure fractional statistics

Motivation:

- Anyons exist in fractional quantum Hall effect (2D electrons in a magnetic field) [1]



Exchange quasiparticles $\Longrightarrow$ wavefunction gets phase $\phi_s$ that is neither 0 (bosons) nor $\pi$ (fermions)

- Simplest bosonic analog: $\nu = 1/2$ Laughlin state

$$\Phi_N(z_1,\ldots,z_N) \propto \prod_{j<k}(z_j - z_k)^2\, e^{-\sum_i |z_i|^2/2}$$

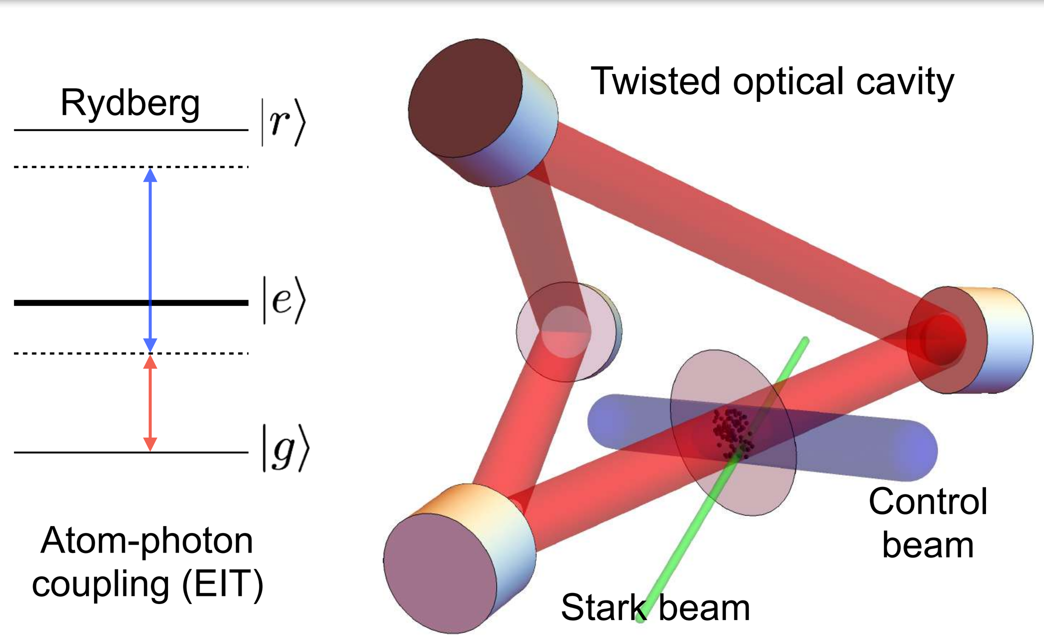
where $z_j = r_j e^{i\theta_j}/l$ is location of $j^{\text{th}}$ particle in the plane ($l$ is magnetic length)

State with quasiholes at $\pm z_0$:

$$\mathrm{QH}_N(\{z_i\}) \propto \prod_j (z_j - z_0)(z_j + z_0)\,\Phi_N(\{z_i\})$$

- Q: How to create & how to measure phase?

## II. Envisioned experimental set-up




- Non-planar geometry realizes effective magnetic field for transverse motion of photons (red) [2]
- Concave mirrors yield transverse harmonic trap
- Photons couple with atoms in a transverse plane under EIT, forming long-lived, strongly interacting polaritons [3,4]
- Polaritons behave as massive interacting bosons in a uniform magnetic field $\Longrightarrow$ can form quantum Hall states
- Extra lasers (Stark beam) can yield localized potentials for creating holes by ac Stark shift [5]

## III. Model

- We model polariton dynamics by a Hamiltonian with contact interaction

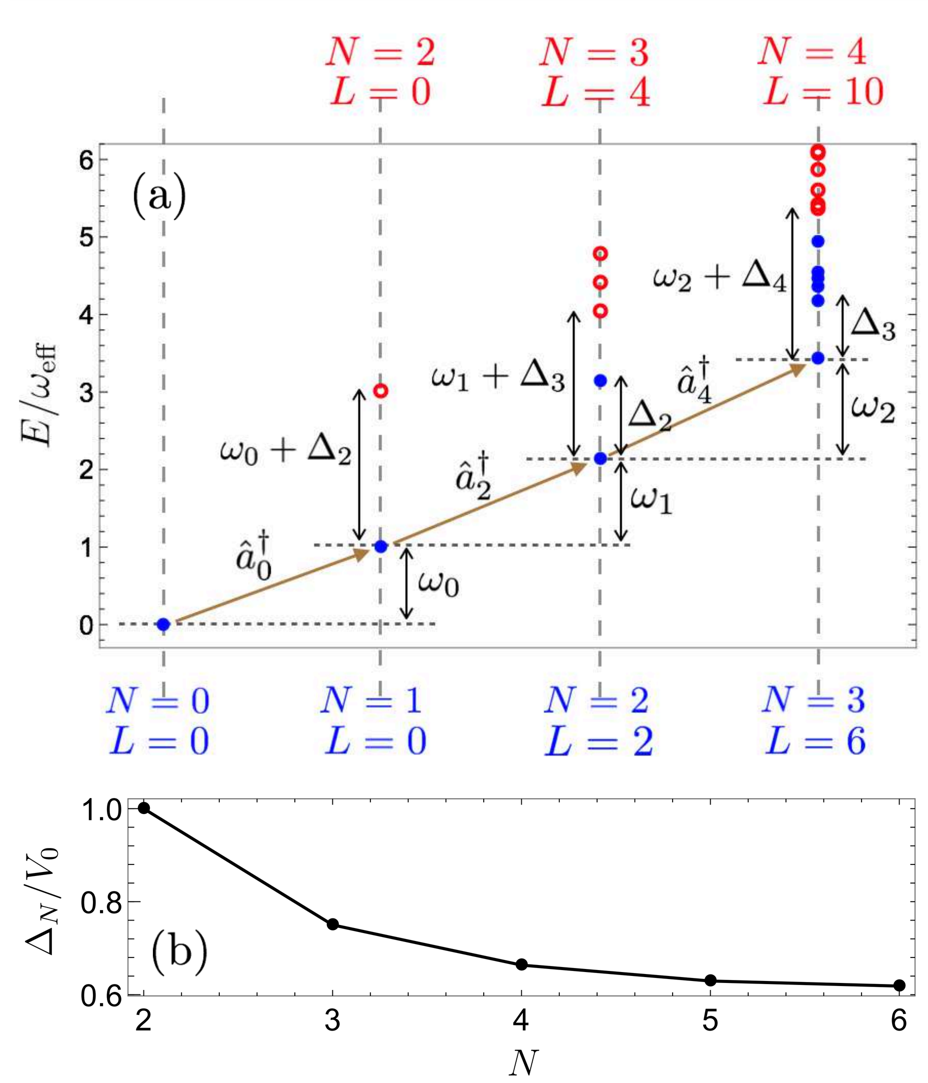
$$\hat{H} = \int d^2r\, \hat{\psi}^\dagger \left[\frac{(-i\vec{\nabla} - M\omega_B r\hat{\theta})^2}{2M} + \frac{1}{2}M\omega_T^2 r^2\right]\hat{\psi} + g\,\hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}\,, \quad \hat{\psi} \equiv \hat{\psi}(\vec{r})$$

- Lowest Landau level spanned by angular momentum eigenstates $\phi_m(z) \propto z^m e^{-|z|^2/2}$ with energy $\omega_{\text{eff}} + m\varepsilon$ $[\omega_{\text{eff}} = (\omega_B^2 + \omega_T^2)^{1/2},\ \varepsilon = \omega_{\text{eff}} - \omega_B]$
- Laughlin state $|\Phi_N\rangle$ is exact N-particle ground state with net angular momentum L=N(N-1)
- Energy scales: (i) Landau level separation $2\omega_B$ (ii) trap-induced splitting $\varepsilon$ (iii) two-particle interaction energy $V_0 = g/(\pi l^2)$ (iv) polariton decay rate $\gamma$
- Typically, $\varepsilon, V_0, \gamma \ll \omega_B$ in experiments $\Longrightarrow$ dynamics confined to lowest Landau level

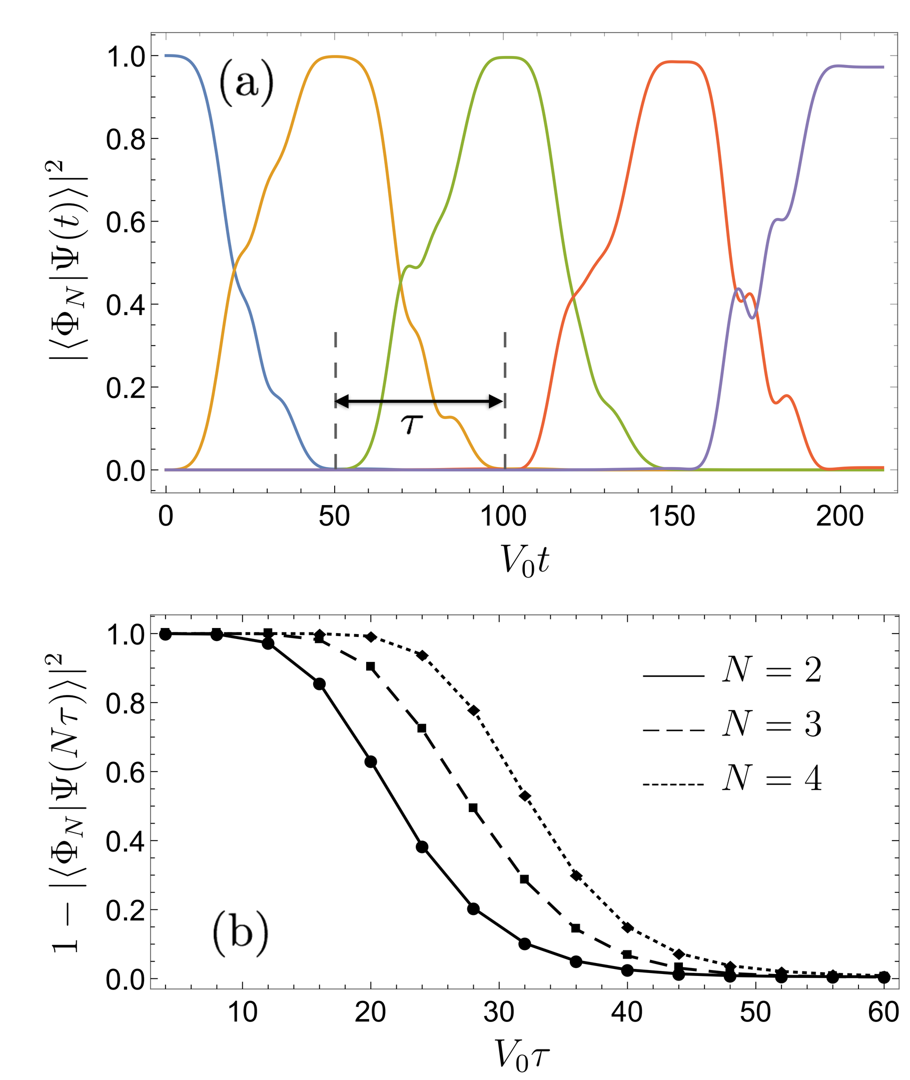
## IV. Creating Laughlin states

Idea: use rapid adiabatic passage to sequentially add polaritons, coherently driving the system through successive Laughlin states of increasing particle number

Steps: (i) pump in angular momentum channel m=2n to couple the states $|\Phi_n\rangle$ and $|\Phi_{n+1}\rangle$ (ii) sweep the drive frequency through resonance to induce transition from n to n+1




(a) Spectrum of states that are coupled during driving. Desired transitions between Laughlin states are shown with (brown) arrows. $\hat{a}_m^\dagger$ adds a particle with angular momentum m. Energy splittings are labeled.
(b) Number dependence of excitation gap $\Delta_N$.



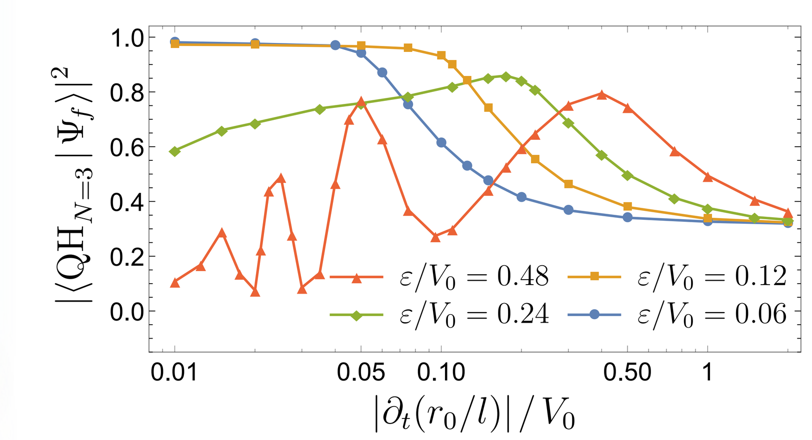

(a) Overlap of system wavefunction $|\Psi(t)\rangle$ with each N-particle Laughlin state as a function of time during drive, for sweep duration $\tau = 50/V_0$. Each successive plateau corresponds to increasing N by 1.
(b) Fidelity of state preparation vs sweep duration.

- Adiabaticity: sweep duration be large compared to excitation gap: $\tau \gg 1/V_0$
- Time to prepare N-particle Laughlin state with high fidelity, $T \gtrsim 50N/V_0$
- Coherence: polariton loss must be small $\Longrightarrow$ $N\gamma T/2 \ll 1$ or $V_0/\gamma \gg 25N^2$ (in current experiments, $V_0/\gamma \sim 50$)

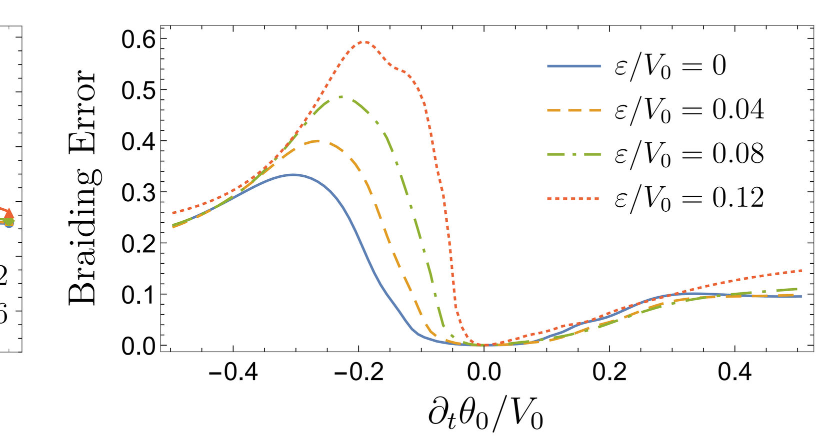
## V. Creating and braiding quasiholes

Bring strong localized potentials into Laughlin cloud through the edge to bind quasiholes, then rotate those potentials to perform braiding

- A small but finite trap frequency required to prevent edge excitations
- Counterclockwise and clockwise braiding are not equivalent as magnetic field breaks time-reversal symmetry




Fidelity of quasihole preparation by sweeping strong delta potentials, vs sweep rate.

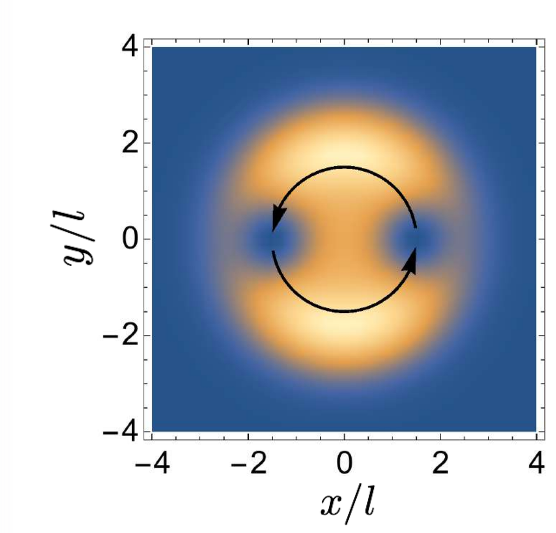



Braiding error, or the time-averaged overlap with excited-state manifold, vs rotation speed.

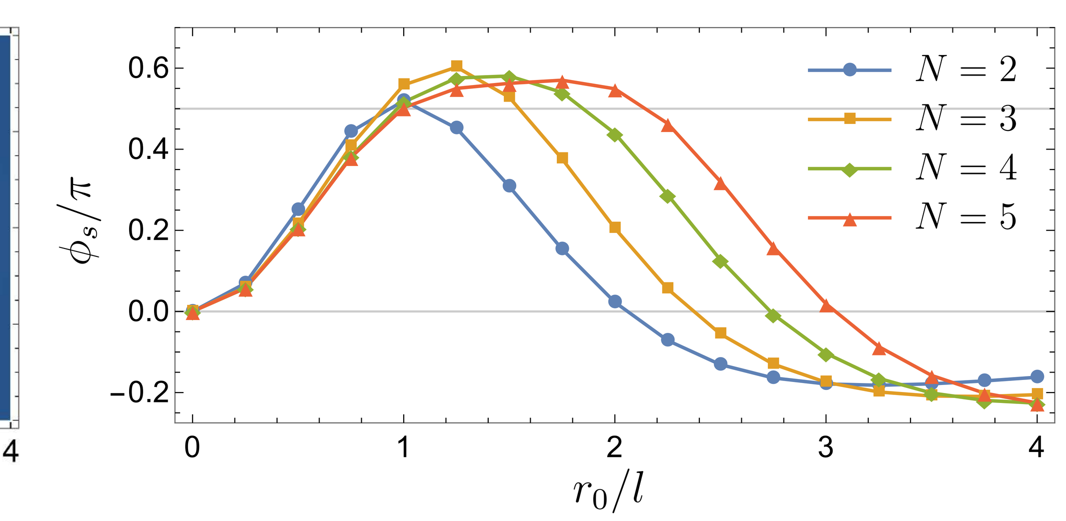
## VI. Measuring fractional statistics

Q: Berry phase acquired during braiding contains both Aharonov-Bohm phase $\phi_{AB}$ and statistical phase $\phi_s$. How to isolate $\phi_s$?

A: Take difference between rotating two holes by $\pi$ and one hole by $2\pi$




Two quasiholes being braided




Statistical phase reaches plateau near $\pi/2$ for large N

Q: How to measure Berry phase?

1. Prepare atoms in a superposition of ground state and a Rydberg state with large blockade radius: $|0\rangle \to |0\rangle + |R\rangle$
2. Create Laughlin state, create and braid quasiholes, remove quasiholes, retrace driving sequence to remove polaritons
3. During this cycle, $|R\rangle$ is unaffected and $|0\rangle$ gains a dynamical phase $\phi_d$ and Berry phase $\phi_B$: $|0\rangle + |R\rangle \to e^{i(\phi_d+\phi_B)}|0\rangle + |R\rangle$
4. Apply $\pi/2$-pulse + measure occupation to extract $\phi_d + \phi_B$
5. Repeat experiment at different rates to eliminate $\phi_d$


1. D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. 53, 722 (1984)
2. N. Schine et. al., Nature (London) 534, 671 (2016)
3. J. Ningyuan et. al., Phys. Rev. A 93, 041802 (2016)
4. N. Jia et. al., arXiv:1705.07475 (2017)
5. L. Li, Y. O. Dudin, and A. Kuzmich, Nature (London) 498, 466 (2013)