Critical response of a quantum van der Pol oscillator

Shovan Dutta and Nigel Cooper

TCM Group, Cavendish Laboratory, University of Cambridge

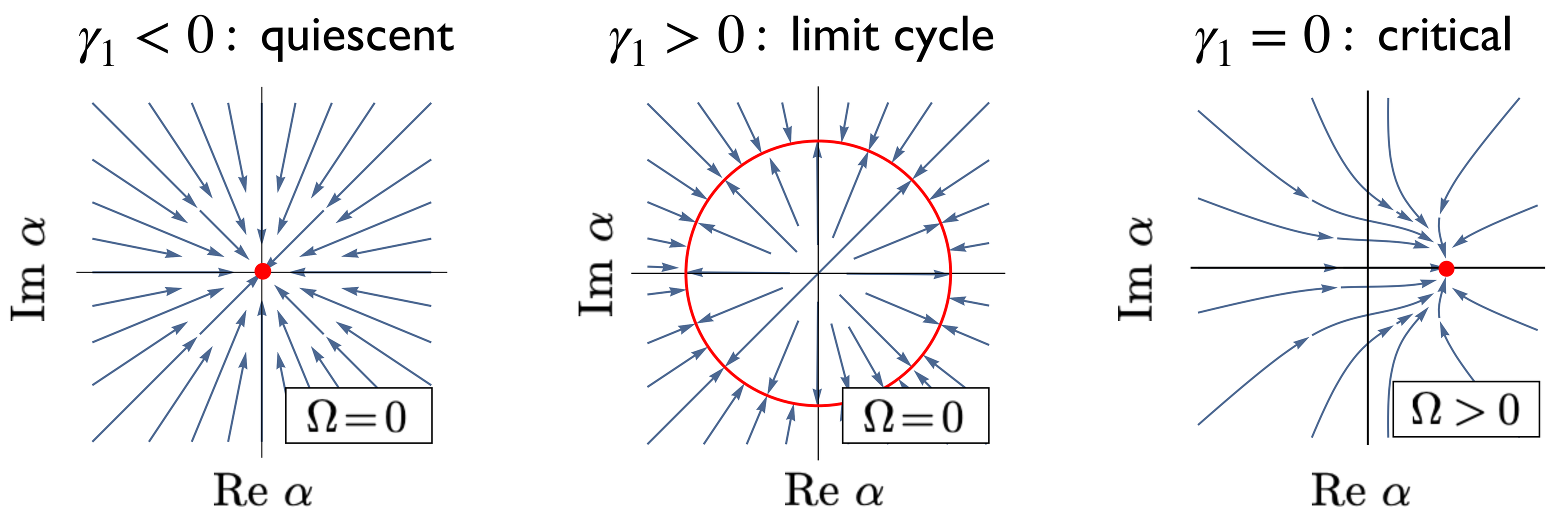
Abstract: Classical nonequilibrium systems close to a dynamical critical point are known to act as efficient sensors. We study the response of such systems in the quantum regime and find new, characteristic features which can be realized in experiments.

I. Classical driven van der Pol

- Iconic model for spontaneous oscillation and synchronization
- Normal form: negative damping, nonlinear damping, & resonant drive

$$\dot{\alpha} = \gamma_1 \alpha - \gamma_2 |\alpha|^2 \alpha + \Omega$$

- Phases at zero drive:

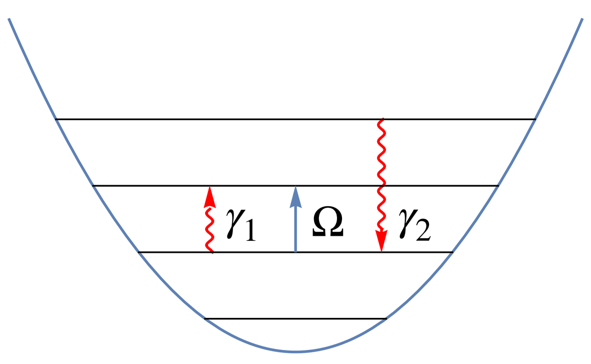


- Critical response fully nonlinear: $\alpha = (\Omega/\gamma_2)^{1/3}$

  ⇒ Infinitely sensitive to weak signals — crucial in hearing [1]
  (enables large dynamic range, frequency tuning,…)

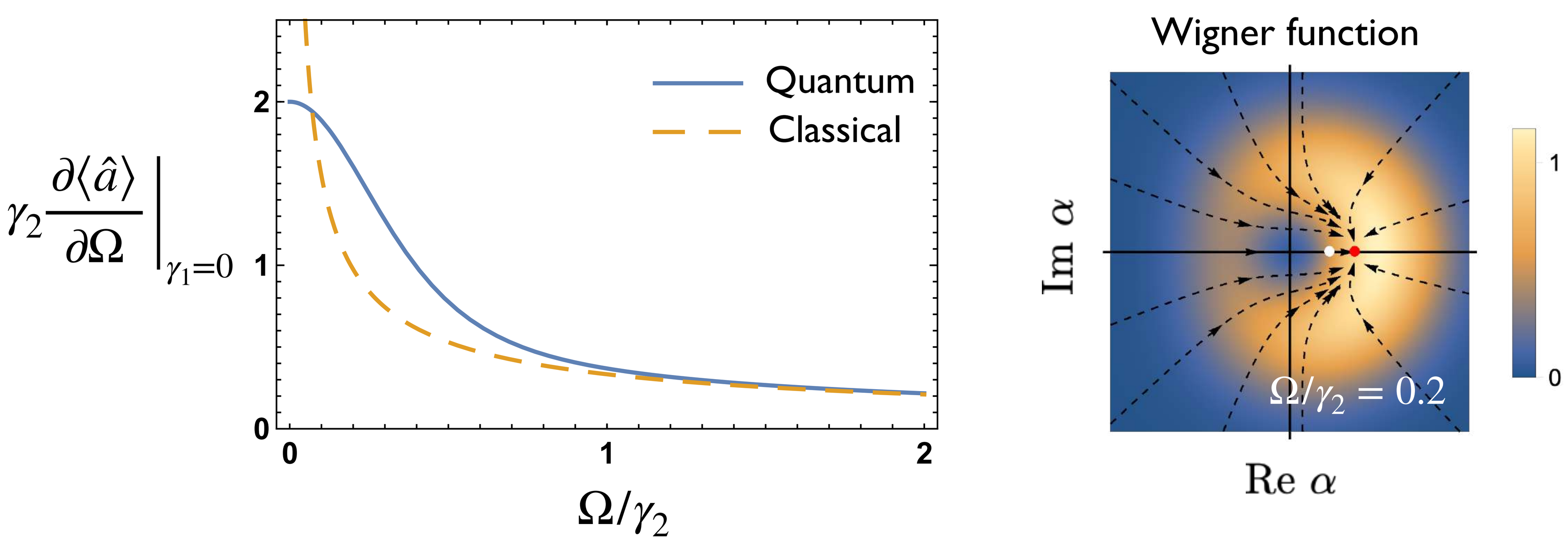
II. Quantum driven van der Pol

- Ingredients: one-particle absorption, two-particle decay, & coherent drive (also realized by state-dependent one-particle dissipation [2])
- Liouvillian dynamics: $\dot{\rho} = \gamma_1 \mathscr{D}[\hat{a}^\dagger]\rho + \gamma_2 \mathscr{D}[\hat{a}^2]\rho + [\Omega(\hat{a}^\dagger - \hat{a}), \rho]$



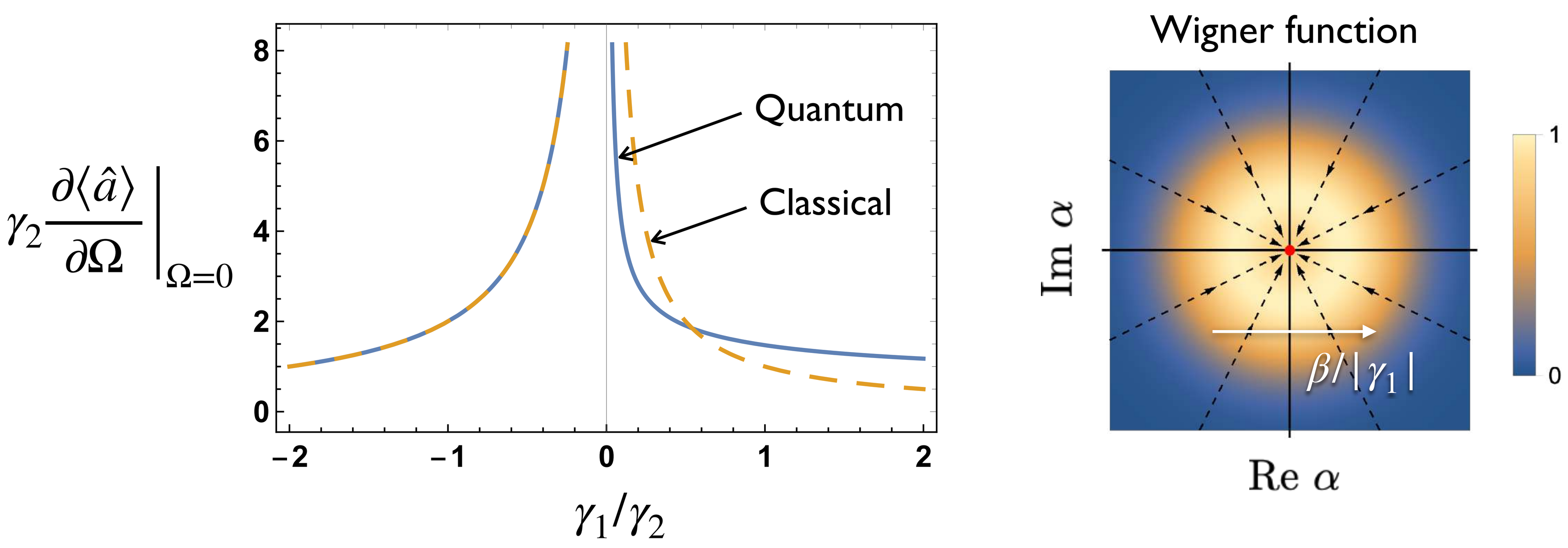
- Driven, open, out of equilibrium
- Simple & experimentally realizable (more in sec. VI)

- Critical response linear below a quantum scale ($\Omega \lesssim \gamma_2$)

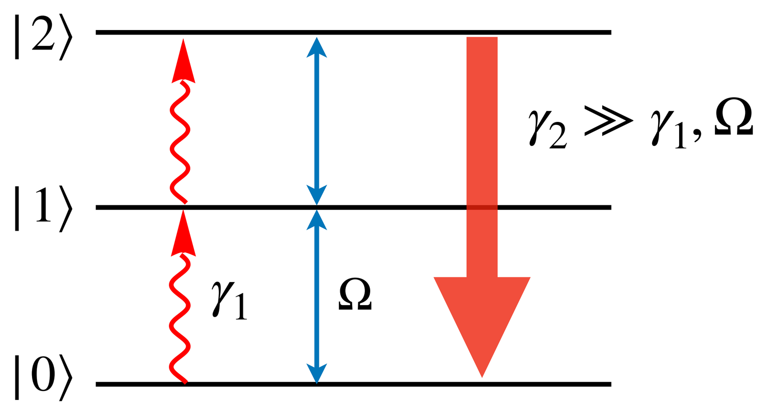


III. Diverging susceptibility

- Zero-field susceptibility still diverges as $1/\gamma_1$ close to critical point!



- Result of blockade physics + competition between drive & dissipation

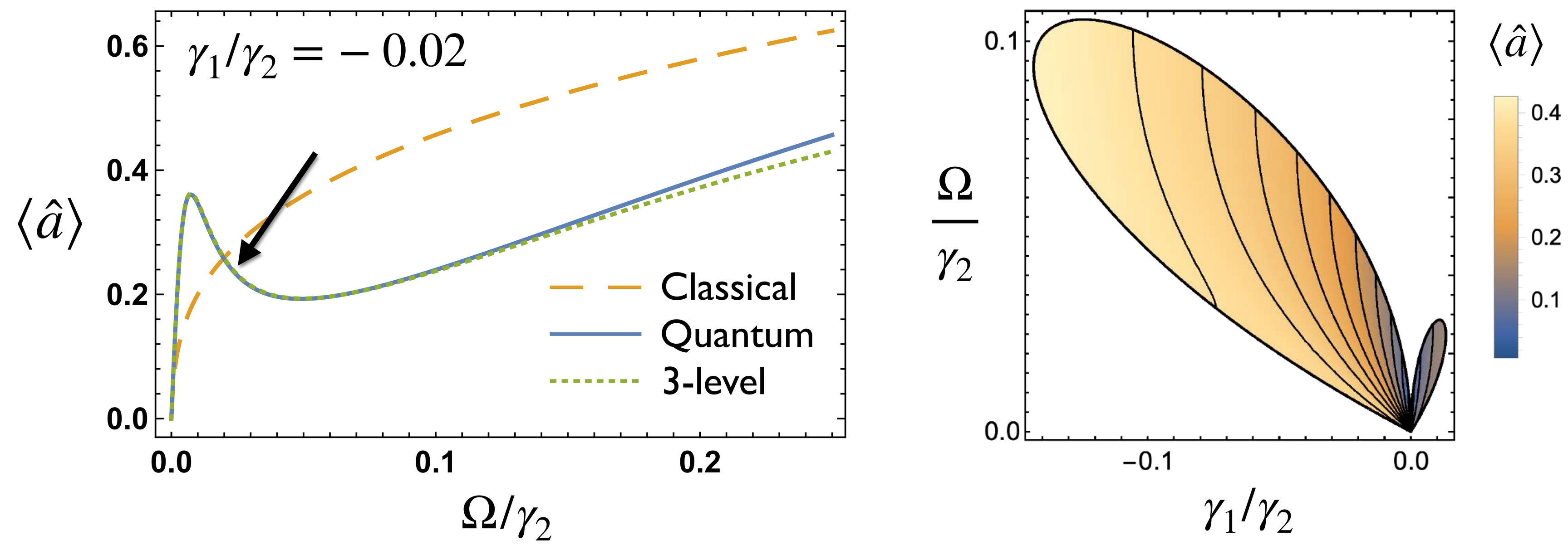


$$\Longrightarrow \quad \langle \hat{a} \rangle \approx \begin{cases} \dfrac{2\Omega}{\gamma_2} + \dfrac{|\gamma_1|}{4\Omega} & \Omega > |\gamma_1| \\ \beta\Omega/|\gamma_1| & \Omega < |\gamma_1| \end{cases}$$

where $\beta = 2\,\Theta(-\gamma_1) + 2/9\,\Theta(\gamma_1)$

IV. Negative susceptibility

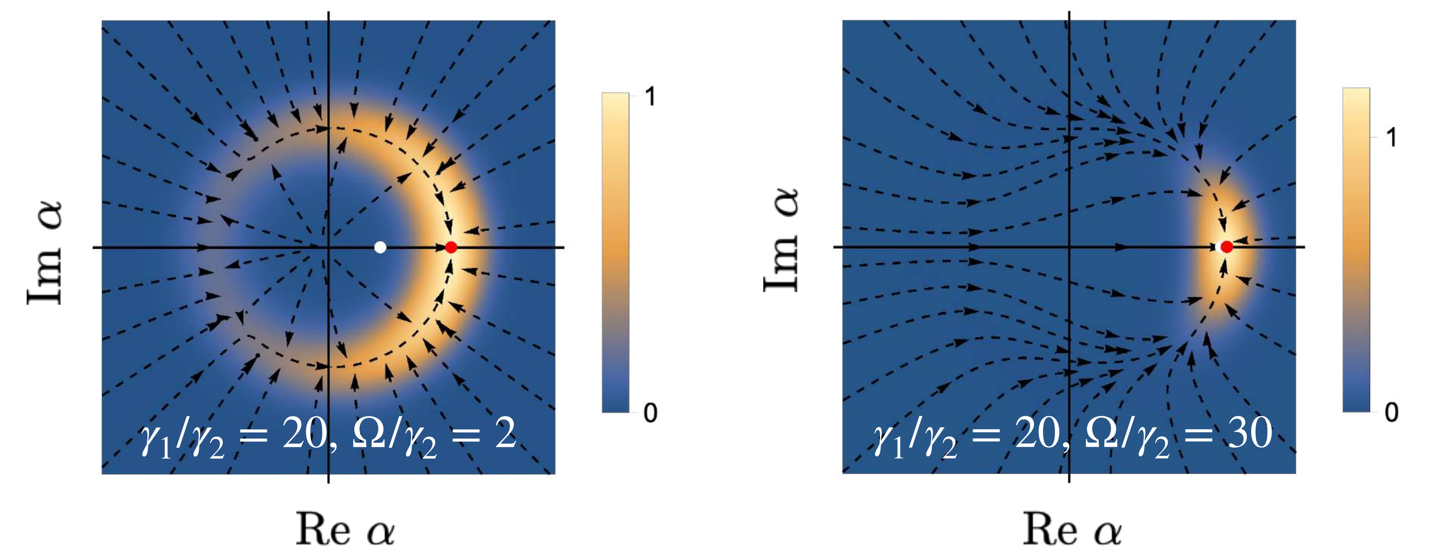
- Coherence decreases with drive close to criticality — purely quantum



- Novel and generic — robust to anharmonicity & detuning

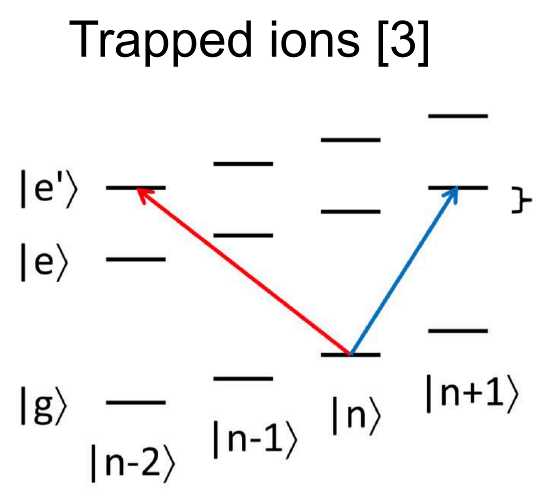
V. Classical limit

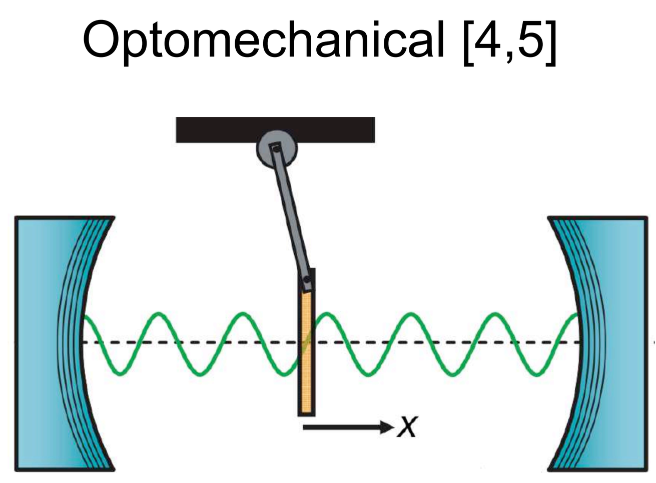
- Classical response recovered in the limit of large drives



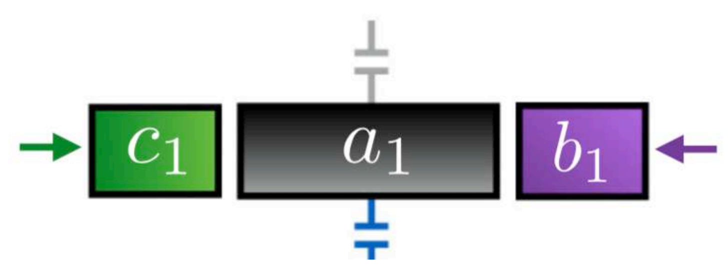
VI. Experimental realization

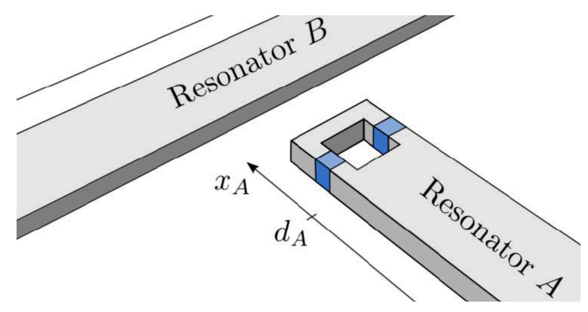
- Motional modes of trapped ions and optomechanical cavities:
  One- and two-particle dissipation by coupling to blue and red sidebands





- Superconducting oscillators (transmons) driven by auxiliary cavities:
  Energy-dependent one-photon transitions by inductive coupling [2,6]





VII. Summary and outlook

- Quantum nonlinear oscillator exhibiting feature-rich dynamical criticality
- Realizable in various setups with one- and two-particle dissipation, or with energy-dependent one-particle dissipation
- Two-particle decay also found in shaken BECs in box trap — CAMEX 1
- Potential application to quantum sensing

1. V. M. Eguíluz *et al.*, PRL 84, 5232 (2000)
2. N. Lorch *et al.*, PRL 118, 243602 (2017)
3. T. E. Lee and H. R. Sadeghpour, PRL 111, 234101 (2013)
4. S. Walter, A. Nunnenkamp, and C. Bruder, PRL 112, 094102 (2014)
5. A. Nunnenkamp *et al.*, PRA 82, 021806(R) (2010)
6. J. R. Johansson, G. Johansson, and F. Nori, PRA 90, 053833 (2014)