Creating and Braiding Anyons in an Optical Cavity

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ARO-MURI Noneq. Dynamics
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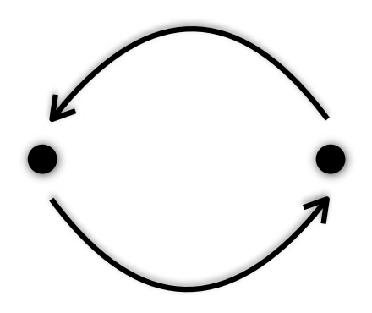
I. Motivation: Anyons

- Fractional exchange statistics — exotic physics in flatland



Bosons: $+1$ Fermions: -1

Anyons: $e^{i\phi_s}$ Non-Abelian: U

- Proposed hardware for fault-tolerant quantum computation [1]
- Anyons exist in fractional quantum Hall effect [2] — hard to observe / control

Q: How to create anyons and measure statistics?

II. Objective

- Create analog of fractional quantum Hall states with light (polaritons)

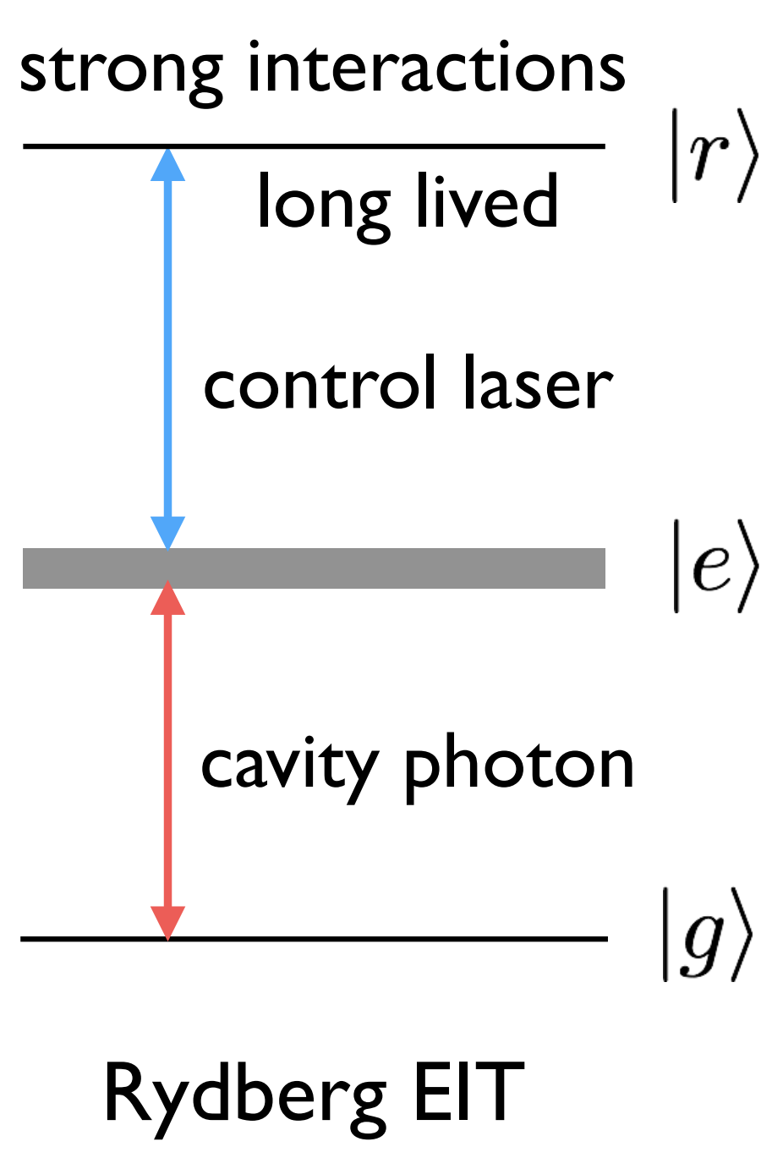
Simplest analog — $\nu = 1/2$ Laughlin state:

$$\Phi_N(z_1,\ldots,z_N) = \prod_{j<k}(z_j - z_k)^2\, e^{-\sum_i |z_i|^2/2}$$

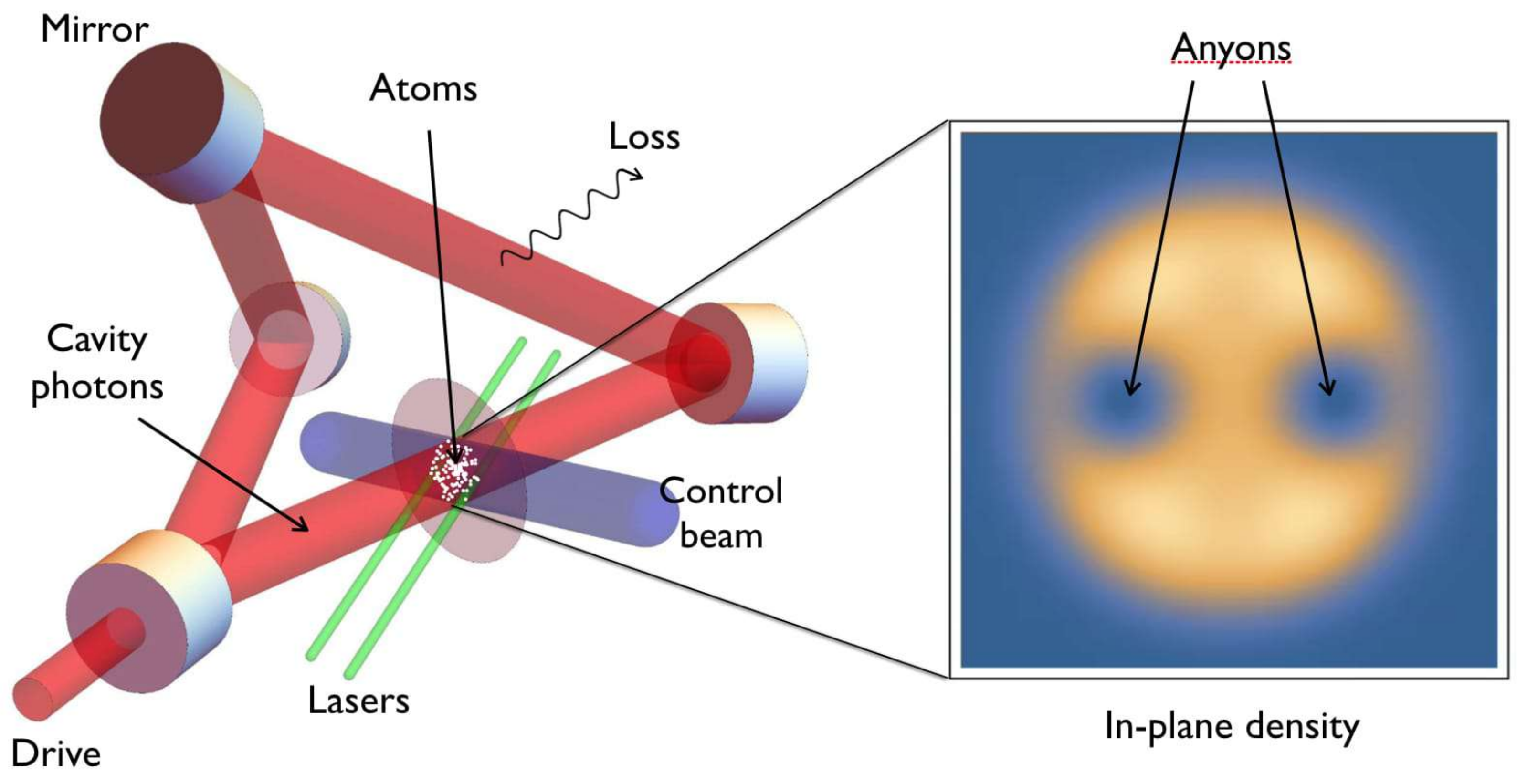
where $z_j \equiv (x_j + iy_j)/l$: location of j^{th} particle, l: magnetic length

- Create anyonic "quasihole" excitations $\Phi_N^{\circ\circ} = \prod_j (z_j - z_0)(z_j + z_0)\Phi_N$
- Braid (exchange) quasiholes and measure statistics

III. Envisioned experiment



Rydberg EIT



- Near-degenerate cavity — longitudinal mode fixed
  ⇒ effective 2D dynamics in transverse plane
- Concave mirrors confine light ⇒ transverse harmonic trap
- Nonplanar geometry rotates light ⇒ effective magnetic field [3]
- Photons couple to atoms under EIT ⇒ long-lived polaritons [4]

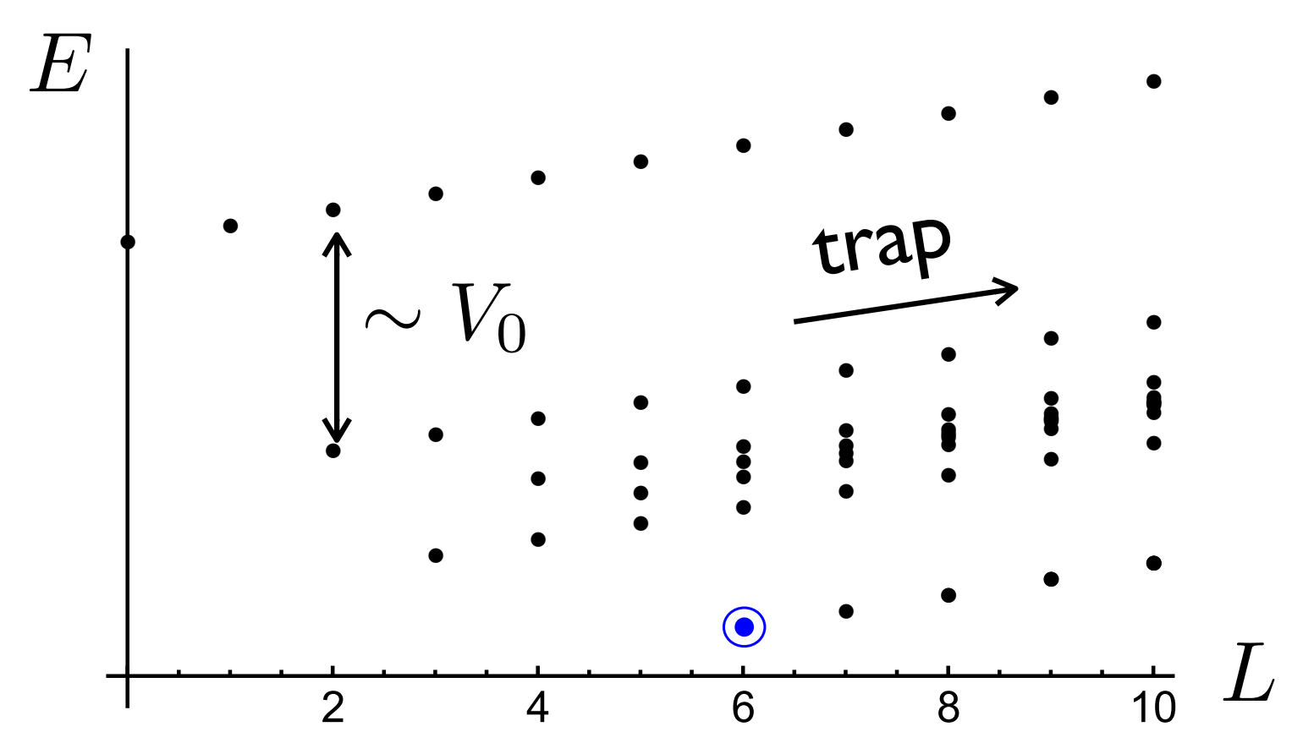
Rydberg polaritons ⟨ Photon dynamics / Rydberg interactions

- Extra lasers yield localized potentials for binding holes

Model: $$\hat{H} = \int d^2r\,\hat\psi^\dagger\Bigg[\underbrace{\frac{(-i\vec\nabla - M\omega_B r\hat\varphi)^2}{2M}}_{\text{kinetic}} + \underbrace{\frac{1}{2}M\omega_T^2 r^2}_{\text{trap}}\Bigg]\hat\psi + \underbrace{\frac{\pi V_0}{2M\omega_B}\hat\psi^\dagger\hat\psi^\dagger\hat\psi\hat\psi}_{\text{interaction}} + \underbrace{F(\vec r,t)\hat\psi^\dagger + F^*(\vec r,t)\hat\psi}_{\text{drive}} + \underbrace{U(\vec r,t)\hat\psi^\dagger\hat\psi}_{\text{potential}}$$

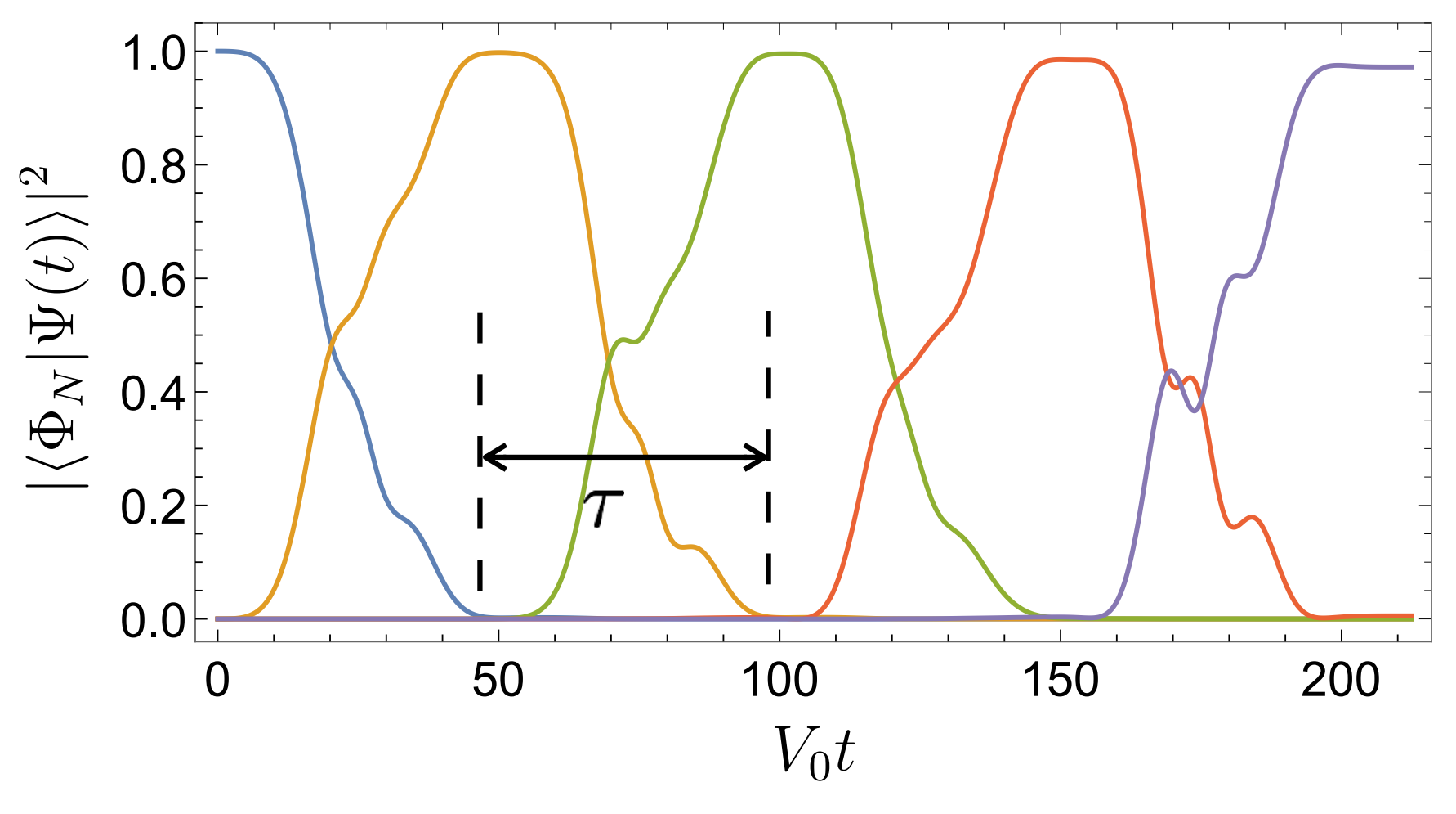
- Energy scales: (1) Landau level spacing $2\omega_B$, (2) trap frequency ω_T, (3) interaction energy V_0, (4) polariton decay rate γ
- $\omega_T, V_0, \gamma \ll \omega_B$ ⇒ Dynamics confined to lowest Landau level, spanned by angular momentum states, $\phi_m(z) = z^m e^{-|z|^2/2}$
- Laughlin state $|\Phi_N\rangle$ is N-particle ground state, with angular momentum $L_N = N(N-1)$ and energy $E_N \approx N\omega_B + N^2\omega_T^2/(2\omega_B)$

Many-body spectrum for N=3

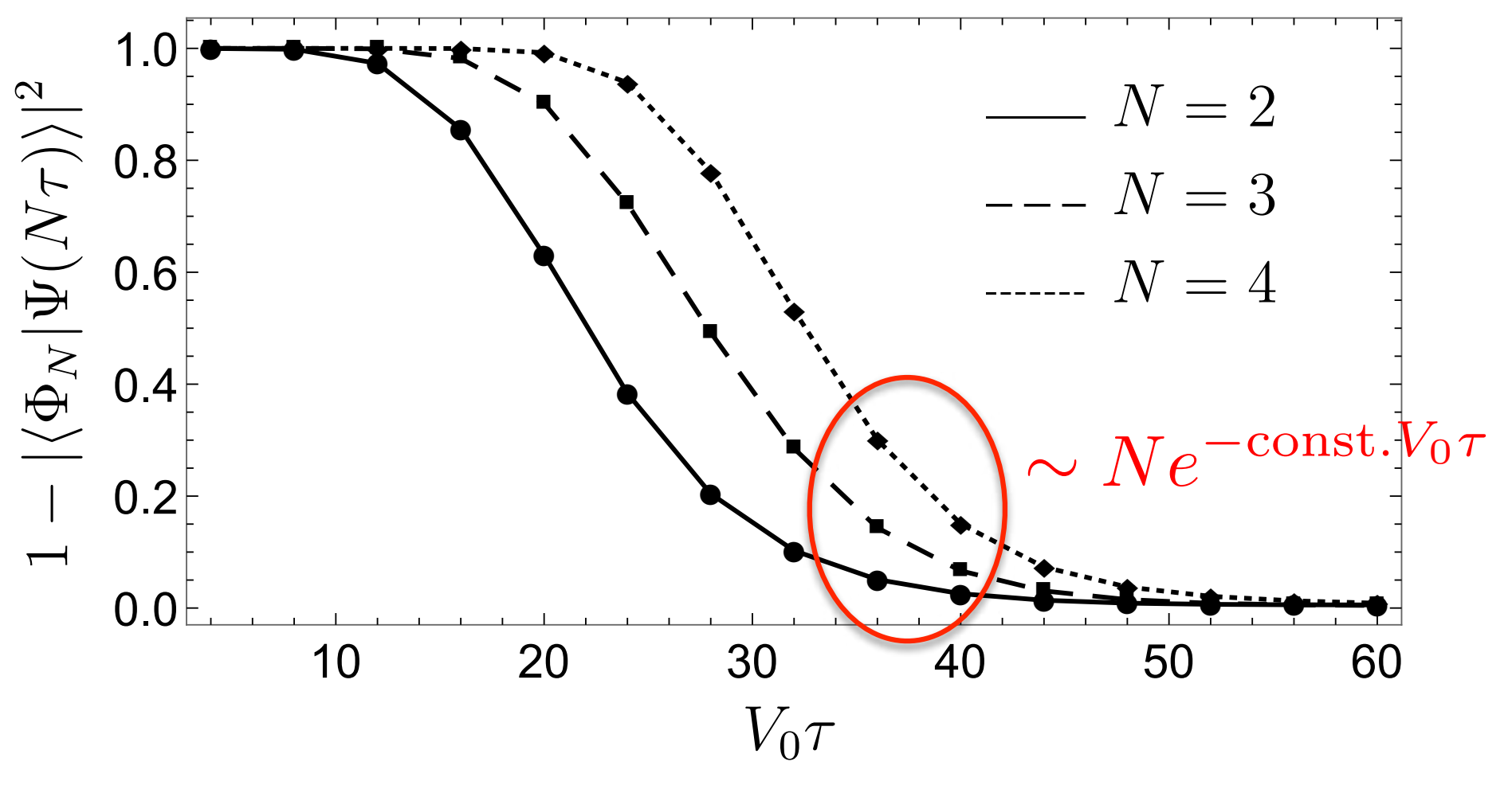


IV. Creating Laughlin state

- Rapid adiabatic passage: inject photons one-by-one such that $|\Phi_0\rangle \to |\Phi_1\rangle \to \cdots \to |\Phi_N\rangle$
  1. Pump photons with angular momentum $m = L_{n+1} - L_n = 2n$ (Laguerre-Gauss)
  2. Sweep drive frequency thru resonance to induce transition $|\Phi_n\rangle \to \Phi_{n+1}\rangle$



Overlap with each N-particle Laughlin state as a function of time, for sweep duration $\tau = 50/V_0$. Each successive plateau corresponds to increasing N by 1.



Cumulative error in state preparation as a function of sweep duration τ for different final particle numbers N. It falls off exponentially for large τ.

- Adiabaticity: sweeps must be slow enough to prevent unwanted excitations ($\tau \gg 1/V_0$)
- Coherence: sweeps must be fast enough to prevent loss ($\tau \ll 1/\gamma$)

Figure of merit $V_0/\gamma \gtrsim 10N^2 \ln N$ (current experiments: $V_0/\gamma \sim 50$)

V. Creating and exchanging quasiholes

- Adiabatically insert strongly focused lasers to bind quasiholes

$$U(\vec r,t) = U_0\big[\delta(\vec r - \vec r_0(t)) + \delta(\vec r + \vec r_0(t))\big], \qquad U_0 \gg V_0/(M\omega_B)$$

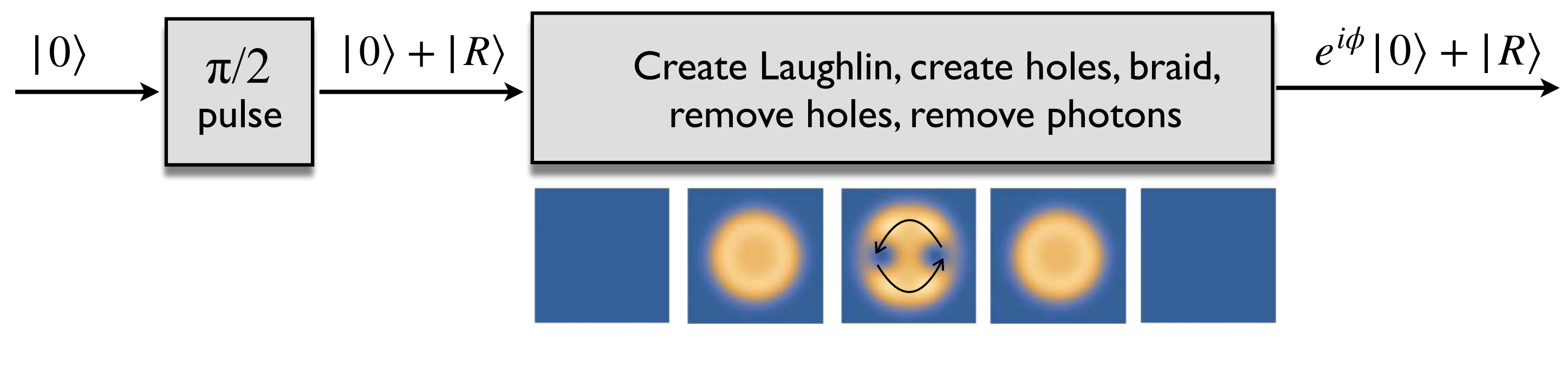
— A small but finite trap suppresses edge excitations

- Use the same lasers to drag quasiholes around one another to perform exchange
- Much faster than creating Laughlin state — figure of merit unaffected

VI. Measuring anyonic statistics

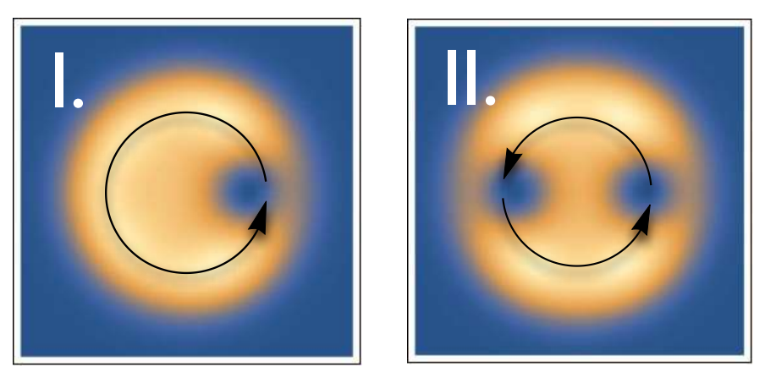
Total phase $\phi = \underbrace{\phi_d}_{\text{dynamical}} + \underbrace{\phi_g}_{\text{geometric}}$ $\phi_g = \underbrace{\phi_{\text{AB}}}_{\text{Aharonov-Bohm}} + \underbrace{\phi_s}_{\text{statistical}}$

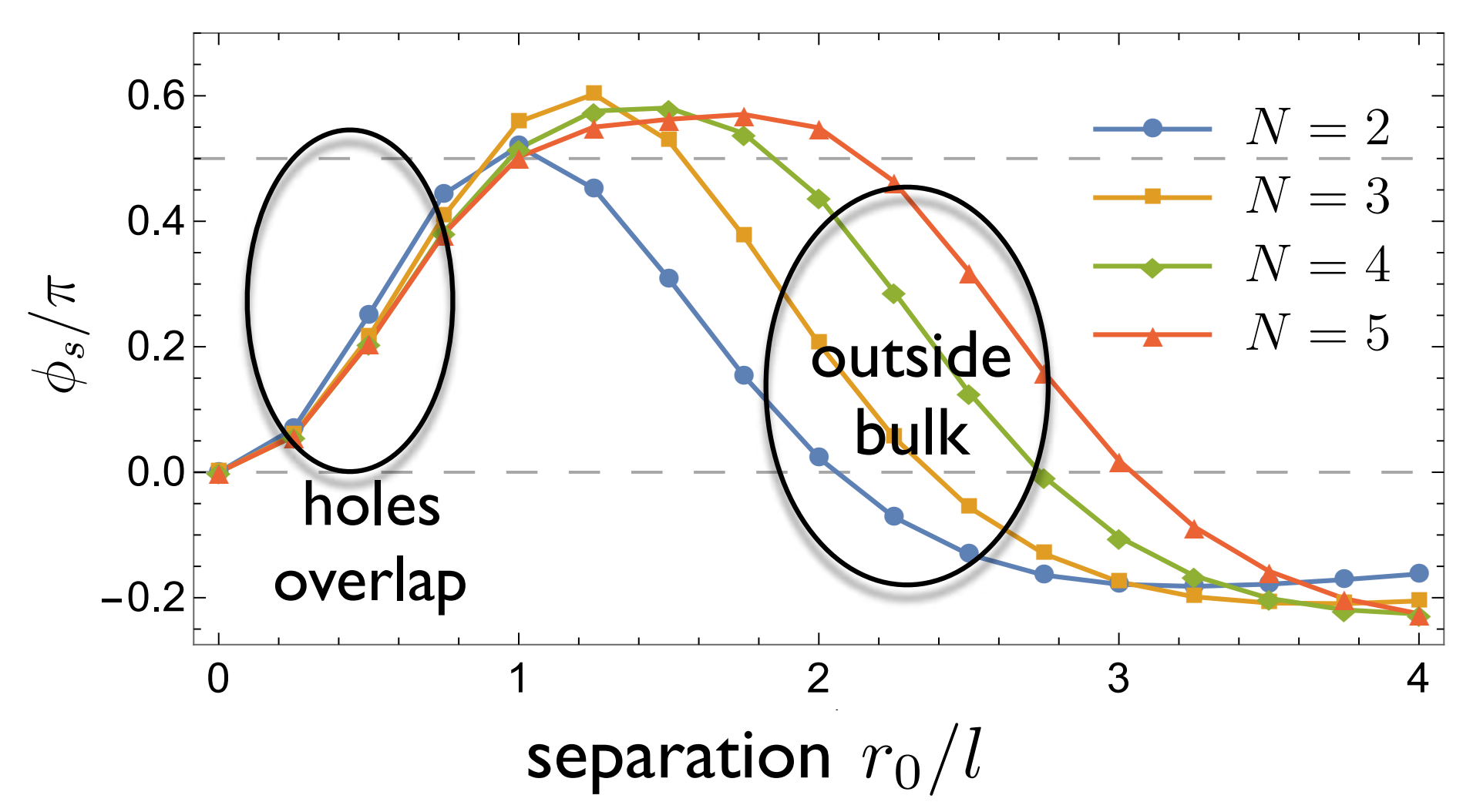
- Measuring ϕ: compare with a reference $|R\rangle$ that is unaffected by drives (e.g., a Rydberg excitation with large blockade radius)



- Repeat experiment at different rates to separate ϕ_g from ϕ_d
- To measure ϕ_s, compare ϕ_g from two experiments:

I. Rotate a single quasihole by 2π: $\phi_g^{\text{I}} = \phi_{\text{AB}}$
II. Rotate two quasiholes by π: $\phi_g^{\text{II}} = \phi_{\text{AB}} + \phi_s$





Distinct plateau emerges near thermodynamic limit ($\phi_s = \pi/2$) for $N \gtrsim 5$

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