Critical response of a quantum van der Pol oscillator

Shovan Dutta and Nigel Cooper

TCM Group, Cavendish Laboratory, University of Cambridge

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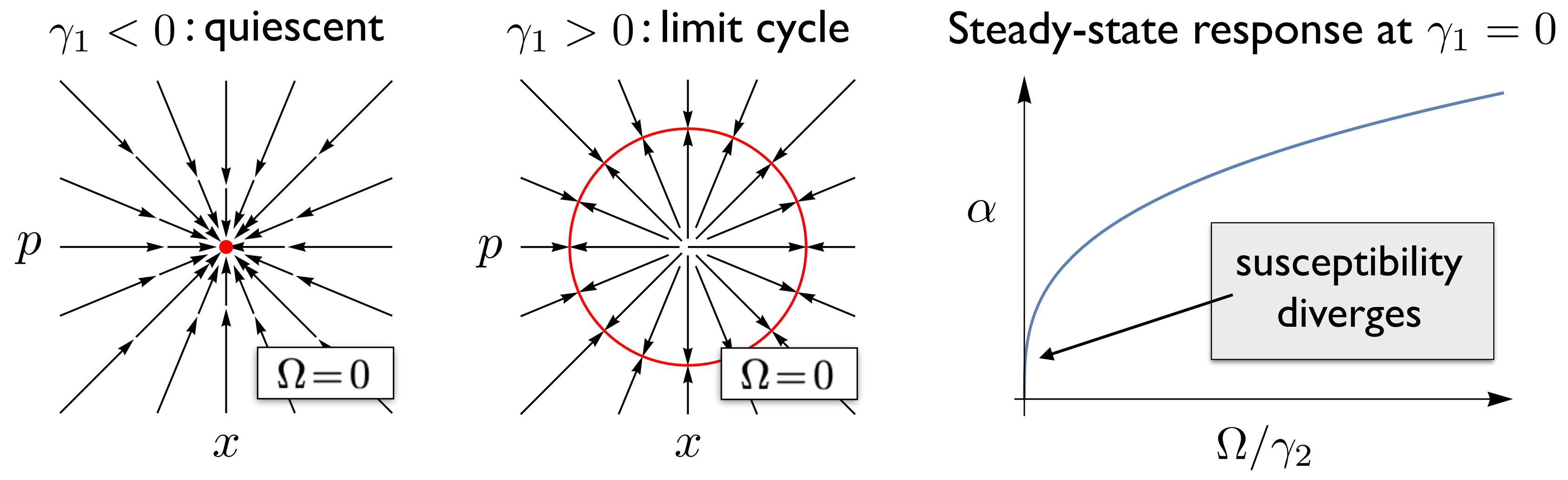
Abstract: Classical dynamical systems close to a critical point are known to act as efficient sensors. We study the response of such systems in the quantum regime and find new, characteristic features which could have important applications to quantum sensing.

I. Classical driven van der Pol

- Iconic model for spontaneous oscillation and synchronization

$$\dot{\alpha} = \underbrace{i\omega_0\alpha}_{\text{natural freq}} \underbrace{+\gamma_1\alpha}_{\text{negative damping}} \underbrace{-\gamma_2|\alpha|^2\alpha}_{\text{nonlinear damping}} \underbrace{+\Omega\, e^{i\omega_0 t}}_{\text{resonant drive}} \qquad [\alpha \equiv x + ip]$$

- Rotating frame: $\dot{\alpha} = \gamma_1\alpha - \gamma_2|\alpha|^2\alpha + \Omega$

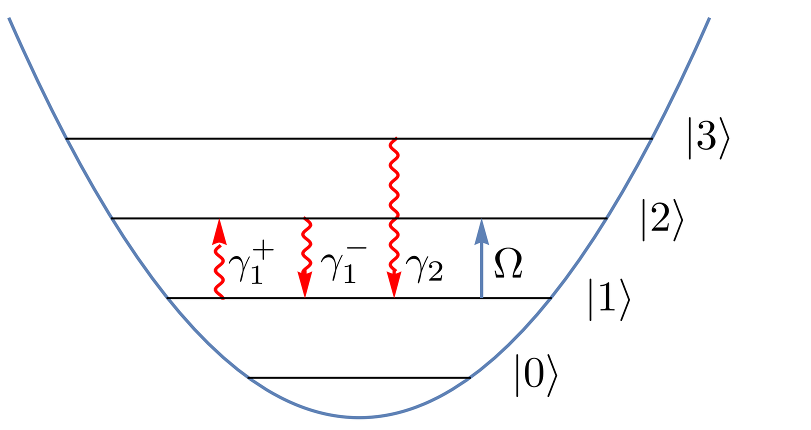


- Infinitely sensitive to weak signals at critical point — crucial in hearing [1]

II. Quantum driven van der Pol

- Harmonic oscillator + drive + dissipation (for implementation, see Sec. VIII)

$$\underbrace{\dot{\hat{\rho}}}_{\text{density matrix}} = \underbrace{\gamma_1^+\mathcal{D}[\hat{a}^\dagger]\hat{\rho}}_{\text{one-body injection}} \underbrace{+\gamma_1^-\mathcal{D}[\hat{a}]\hat{\rho}}_{\text{one-body loss}} \underbrace{+\gamma_2\mathcal{D}[\hat{a}^2]\hat{\rho}}_{\text{two-body loss}} \underbrace{+[\Omega(\hat{a}^\dagger - \hat{a}), \hat{\rho}]}_{\text{coherent drive}}$$



where $\mathcal{D}[\hat{x}]\hat{\rho} \equiv \hat{x}\hat{\rho}\hat{x}^\dagger - \{\hat{x}^\dagger\hat{x}, \hat{\rho}\}/2$.

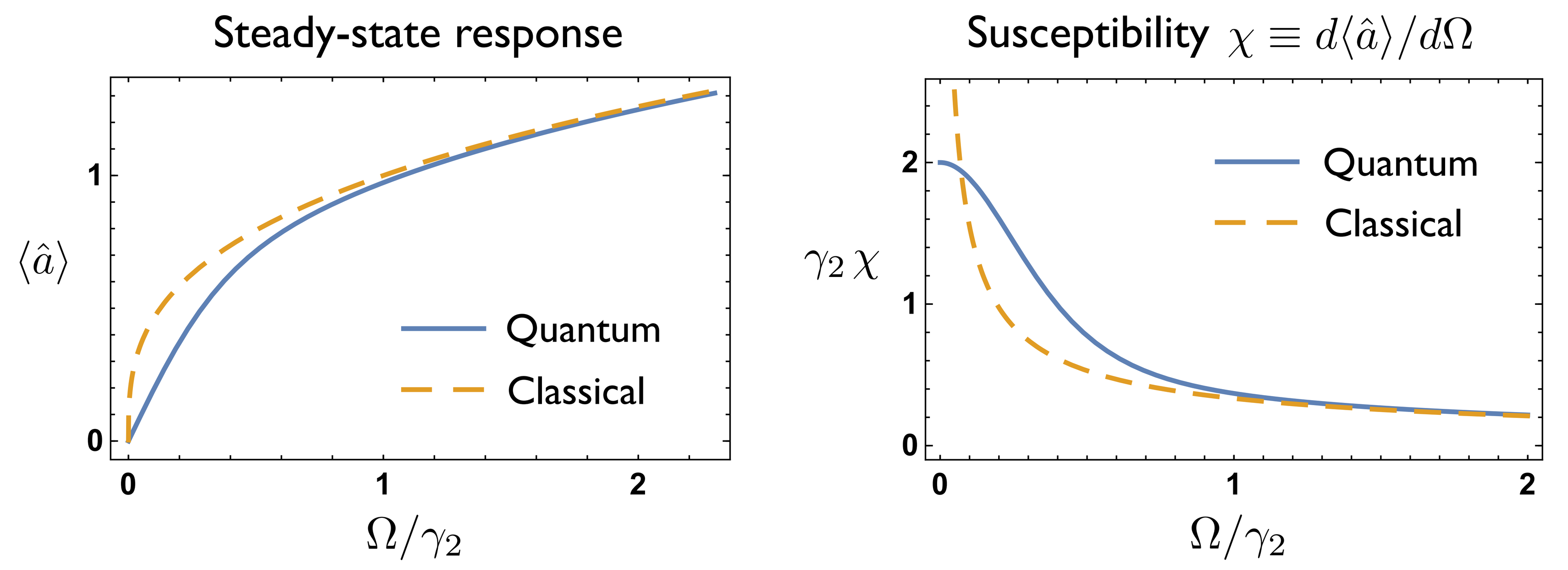
Heisenberg equation of motion:

$\dot{\hat{a}} = \frac{1}{2}(\gamma_1^+ - \gamma_1^-)\hat{a} - \gamma_2\hat{a}^\dagger\hat{a}\hat{a} + \Omega$

⇒ Critical case: $\gamma_1^+ = \gamma_1^-$

III. Critical response

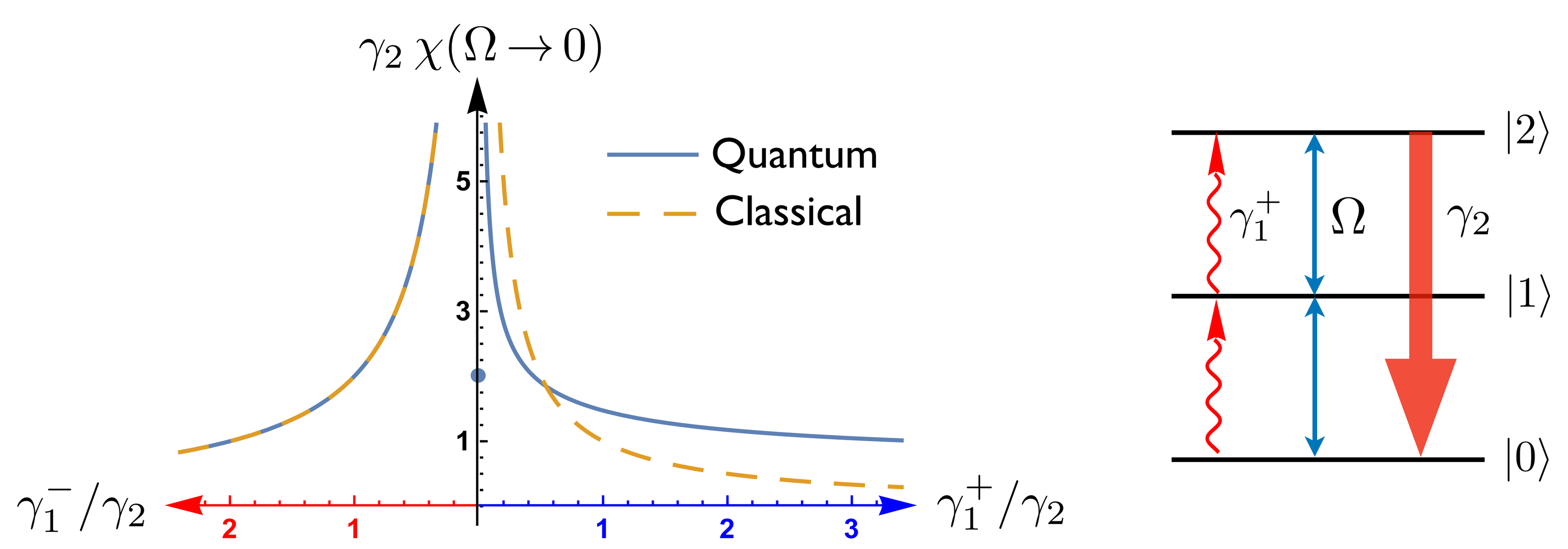
Simplest case: $\gamma_1^+ = \gamma_1^- = 0$



- Classical response persists well into quantum regime, $\langle\hat{a}\rangle \sim 1$
- Divergence cut off at weak drive — finite susceptibility

IV. Diverging susceptibility

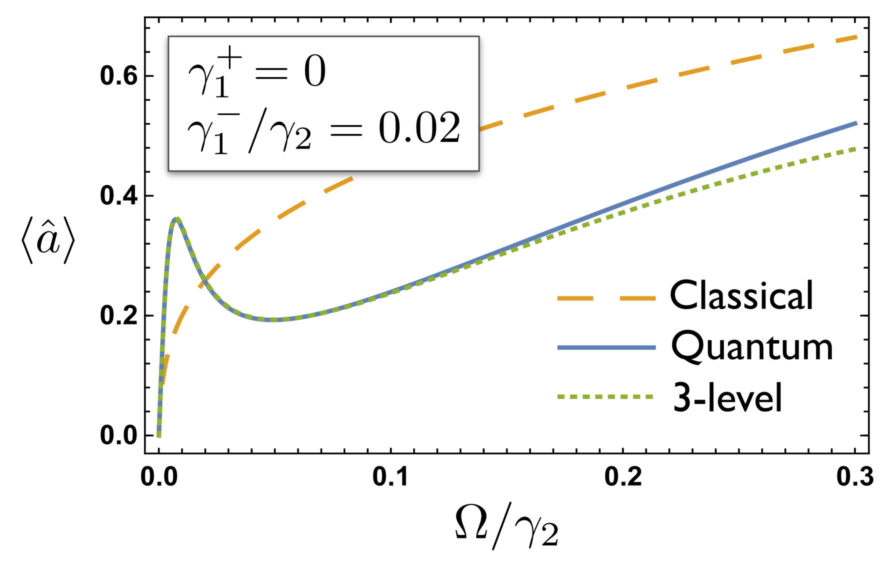
- Yet zero-field susceptibility diverges as $1/\gamma_1^\pm$ for small $\gamma_1^\pm$!



- $\gamma_1^\pm \ll \gamma_2 \Rightarrow$ dynamics projected onto lowest three levels (blockade)
- Competition between drive and dissipation ⇒ coherences grow as $\Omega/\gamma_1^\pm$

V. Nonmonotonic response

- Resolution: response strongly nonlinear for $\Omega \gtrsim \gamma_1^\pm$ — purely quantum



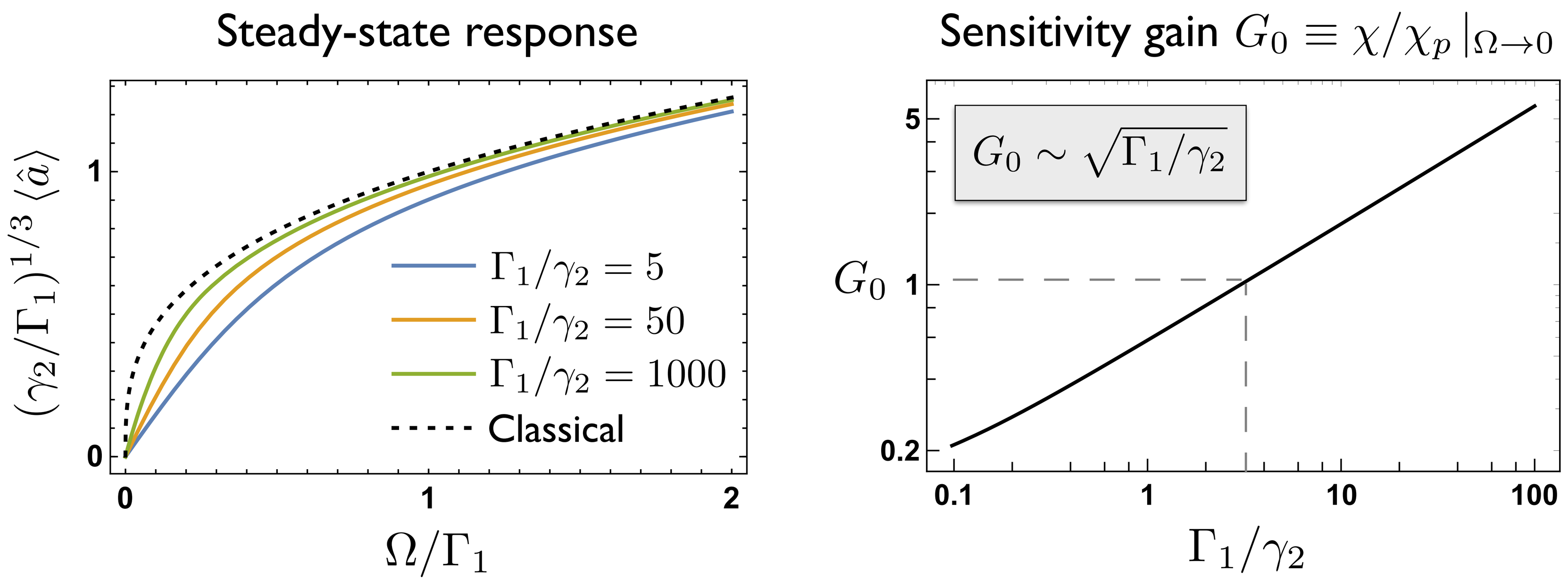
Three-level approximation:

$$\langle\hat{a}\rangle \approx \begin{cases} 2\Omega/\gamma_1^- & \Omega < \gamma_1^- \\ \frac{2\Omega}{\gamma_2} + \frac{\gamma_1^-}{4\Omega} & \Omega > \gamma_1^- \end{cases}$$

- Negative susceptibility for $\gamma_1^\pm \lesssim \Omega \lesssim (\gamma_1^\pm\gamma_2)^{1/2}$ — robust to anharmonicity

VI. Enhanced sensing at criticality

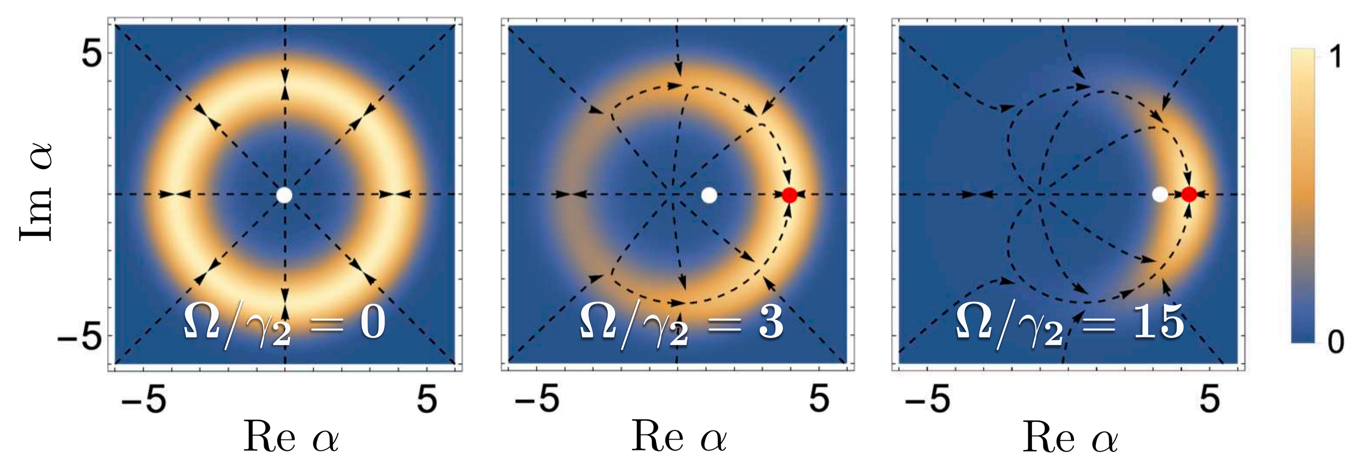
- Damped oscillator has susceptibility $\chi_p = 2/\gamma_1^-$ — can we engineer $\chi > \chi_p$?
- Yes, e.g., set $\gamma_1^+ = \gamma_1^- \equiv \Gamma_1$ and increase Γ_1 to approach classical limit



VII. Sensing limited by two-body loss

- Sensitivity further enhanced by strong particle injection, $\gamma_1^+ \gg \gamma_1^- \gg \gamma_2$
- Drive induces swift transition from limit cycle to classical steady state

Wigner function for $\gamma_1^-/\gamma_2 = 20$ and $\gamma_1^+/\gamma_2 = 50$



- Susceptibility $\chi \approx 2/(3\gamma_2)$, only limited by two-body loss ⇒ $G_0 \approx \gamma_1^-/(3\gamma_2) \gg 1$

VIII. Realization

- Trapped ions, optomechanical membranes, superconducting resonators — drive blue/red sidebands to engineer particle injection/loss [2,3].
- Strong two-body loss results from four-wave mixing in circuit QED [4], polariton blockade in cavities [5], and shaken BECs in box traps
- Alternatively, use energy-dependent one-body loss [6]
- Measure response by tomography [7] or direct mapping of Wigner function [8]

IX. Summary

- Prototypical quantum system exhibiting feature-rich dynamical criticality
- Genuine quantum features: diverging susceptibility and nonmonotonic response
- Susceptibility limited only by two-body loss — application to quantum sensing

1. Eguiluz *et al.*, PRL 84, 5232 (2000)
2. Lee and Sadeghpour, PRL 111, 234101 (2013)
3. Walter, Nunnenkamp, and Bruder, PRL 112, 094102 (2014)
4. Leghtas *et al.*, Science 347, 853 (2015)
5. Jia *et al.*, Nat. Phys. 14, 550 (2018)
6. Lörch *et al.*, PRL 118, 243602 (2017)
7. Lvovsky and Raymer, RMP 81, 299 (2009)
8. Lutterbach and Davidovich, PRL 78, 2547 (1997)