Long-range coherence and multistability of lossy qubits

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Abstract: We show that for a simple dissipative drive and loss, a reflection-symmetric qubit array can exhibit multiple highly coherent steady states, including maximally entangled states of nonlocal Bell pairs. We show how to prepare these states in existing setups.

I. Motivation

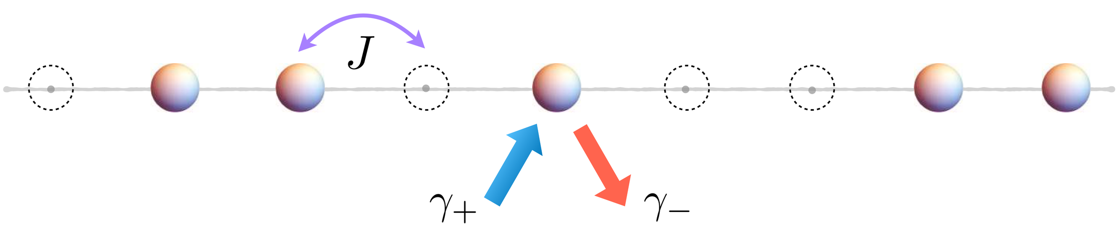
- Stable coherence essential for quantum control and information processing
- Environmental coupling typically destroys coherence — classical steady state
- Coupling can be engineered in synthetic platforms, offering new resource

$\Longrightarrow$ Identify simple lossy setups where coherence can be stabilized

II. Setup and model

Hard-core bosons on 1D lattice with incoherent pump and loss at center

- every site has occupation $n_i = 0$ or 1
- equivalent to spin-1/2 XY chain with spin flip at center



Local pump/loss realized using transmon qubits in microwave circuits [1] and ionizing beams in optical lattices [2]

As we show, center drive yields multiple steady states with long-range coherence

Hamiltonian: $\hat{H} = -J\sum_{i=-l}^{l-1} \hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}$ Jump operators: $\hat{L}_+ = \sqrt{\gamma_+}\,\hat{b}_0^\dagger,\ \hat{L}_- = \sqrt{\gamma_-}\,\hat{b}_0$

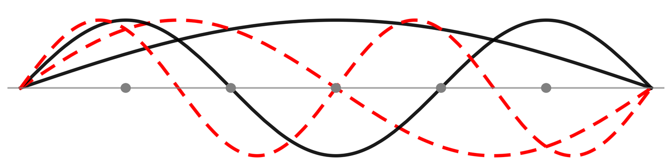
Markovian dynamics [3]: $d\hat{\rho}/dt = -i[\hat{H},\hat{\rho}] + \sum_{\alpha=\pm} \hat{L}_\alpha \hat{\rho} \hat{L}_\alpha^\dagger - \{\hat{L}_\alpha^\dagger \hat{L}_\alpha, \hat{\rho}\}/2$

Fermion description — Jordan-Wigner map: $\hat{f}_j = (-1)^{\sum_{i<j}\hat{n}_i}\hat{b}_j$

$\Longrightarrow$ free-fermion Hamiltonian + nonlocal dissipation (mediates interactions!)

$\downarrow$

even and odd single-particle modes



III. Symmetry and conservation law

Hidden symmetry operator $\hat{C} := -1/2 + \sum_{k=-l}^{l} \hat{f}_k^\dagger \hat{f}_{-k}$ $\quad (\hat{C}^\dagger = \hat{C})$

Symmetry of both Hamiltonian and dissipation:

- $\hat{C} = \hat{N}_{\text{even}} - \hat{N}_{\text{odd}} - 1/2 \implies [\hat{H},\hat{C}] = 0$
- $\hat{C} = (-1)^{\hat{n}_0}\hat{F}(\{\hat{b}_j, \hat{b}_j^\dagger : j \neq 0\}) \implies [\hat{L}_\pm, \hat{C}^2] = 0$

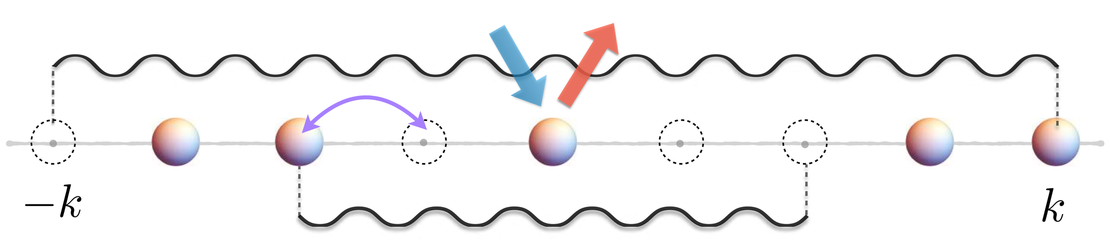
$\Longrightarrow d\langle \hat{C}^2\rangle/dt = 0$, i.e., $\hat{C}^2$ gives a good quantum number

Dynamics decouple into eigenspaces of $\hat{C}^2$ — weight in each sector conserved [4]

IV. Entangled particle-hole pairs

$$\hat{C} = \hat{n}_0 - 1/2 + \underbrace{\sum_{k=1}^{l} (\hat{a}_{k,+}^\dagger \hat{a}_{k,+} - \hat{a}_{k,-}^\dagger \hat{a}_{k,-})}_{\text{Total "charge" } \nu} \quad \text{where} \quad \hat{a}_{k,\pm} := \frac{1}{\sqrt{2}}(\hat{f}_k \pm \hat{f}_{-k})$$

$\hat{a}_{k,\pm}^\dagger|\text{vac}\rangle \sim |0_k 1_{-k}\rangle \pm |1_k 0_{-k}\rangle$ — Bell pair at k and $-k$ with "charge" ± 1



$\hat{a}_{k,+}^\dagger \hat{a}_{k,-}^\dagger|\text{vac}\rangle \sim |1_k 1_{-k}\rangle$ (not entangled) $\Longrightarrow$ net "charge" measures entanglement

$\hat{C}^2$ has eigenvalues $\Lambda = (\nu + n_0 - 1/2)^2,\ \nu = 0, \pm 1, \ldots, \pm l$

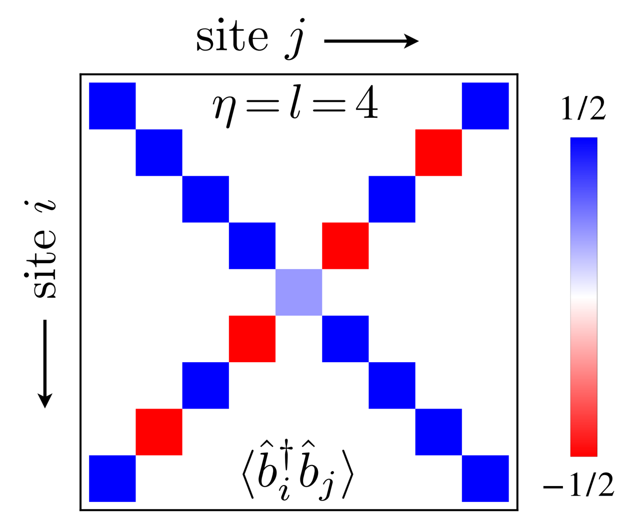
- $l+1$ distinct symmetry sectors $\Lambda \in \{(\eta + 1/2)^2 : \eta = 0, \ldots, l\}$
- Steady state uniquely specified by initial weights $\langle \hat{P}_\eta \rangle$ where $\hat{P}_\eta$ is the projector onto sector η

$\eta = 0$: minimally entangled $\qquad \eta = l$: maximally entangled

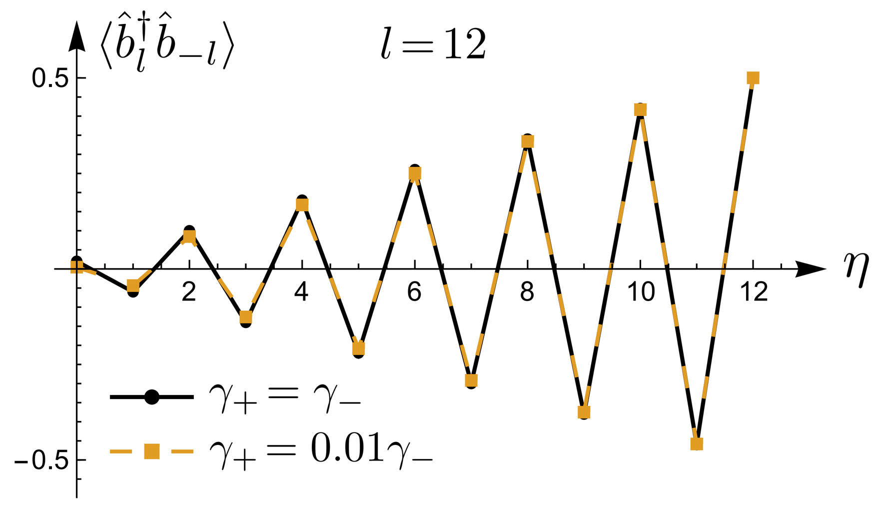
V. Steady states

Steady state in sector η: $\hat{\rho}_\eta = (\gamma_+/\gamma_-)^{\hat{N}}\hat{P}_\eta$ (normalized) $\quad \hat{N}$: total occupation

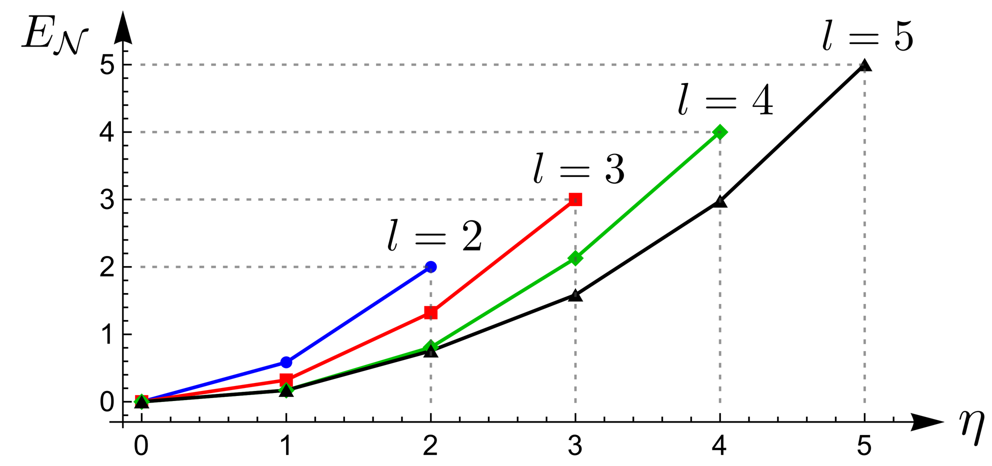
- infinite-temperature state w/ chemical potential $\mu = \ln(\gamma_+/\gamma_-)$
- but all states in a sector can be highly entangled! [e.g., $\eta = l$ has Bell pairs of same "charge" at all positions]



Maximally entangled sector



End-to-end coherence in different sectors



Log negativity $E_{\mathcal{N}}$ measuring entanglement between left and right halves (max number of distillable Bell pairs)

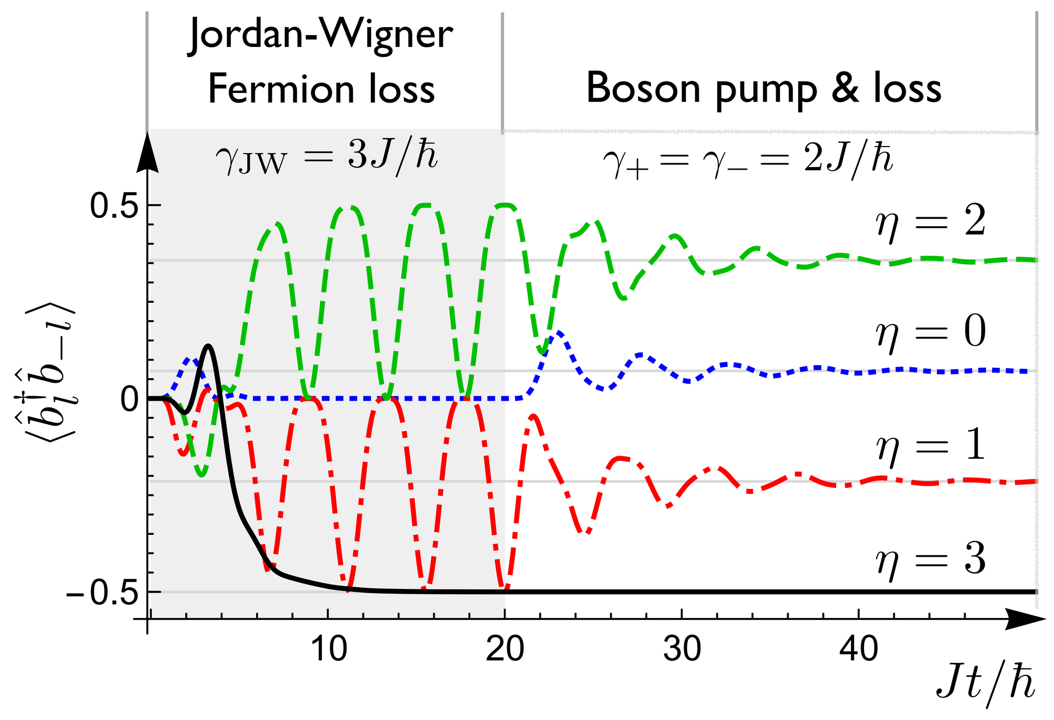
In contrast, for free fermion pump & loss, all odd modes form an exponentially large decoherence-free subspace

VI. Preparing a given sector

Final state $\hat{\rho} = \sum_\eta \langle \hat{P}_\eta \rangle \hat{\rho}_\eta$ — Q: how to prepare initial sector weights $\langle \hat{P}_\eta \rangle$?

Strategy: start with a definite occupation of odd modes (N_{odd}), then deplete all even modes [Recall: $\hat{C} = \hat{N}_{\text{even}} - \hat{N}_{\text{odd}} - 1/2$]

- Engineer loss of Jordan-Wigner fermions at center [5]: $\hat{f}_0 = (-1)^{\sum_{i<0}\hat{n}_i}\hat{b}_0$
  - — wavefunction gains a collective phase when boson is lost
  - — odd modes vanish at center, thus immune to loss
- Prepare symmetric Fock state $\prod_k (\hat{b}_k^\dagger \hat{b}_{-k}^\dagger)^{n_k}|\text{vac}\rangle$ — has $N_{\text{odd}} = \sum_k n_k$
  - — with (only) fermion loss, driven to sector $\eta = \sum_k n_k$
- Switch to boson pump and loss at center — driven to steady state $\hat{\rho}_\eta$



End-to-end coherence during preparation ($l = 3$)

Converges within few tens of tunneling time, typically much faster than residual dissipation and disorder [1]

VII. Summary

- Simple lossy quantum system exhibiting multiple highly coherent nonequilibrium steady states that can be selectively prepared in existing setups
- Controlled generation and preservation of entanglement in an open setting
- Dynamical symmetry stabilizing nonlocal Bell pairs — robust to a large class of two-level 1D systems (w/ symmetric traps)
- Surprising features at other pump-loss configurations — Ref. [6]

1. Ma *et al.*, Nature 566, 51 (2019)
2. Barontini *et al.*, PRL 110, 035302 (2013)
3. Rivas *et al.*, New J. Phys. 12, 113032 (2010)
4. Buča and Prosen, New J. Phys. 14, 073007 (2012)
5. Zhu *et al.*, npj Quantum Inf. 4, 16 (2018)
6. Dutta and Cooper, in preparation