Supplemental Material: "Quantum walk of two anyons across a statistical boundary"

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SI. Characterization of the slow bunched wave

In this section we investigate the effect of the exchange statistics on the speed and relative weight of the slow bunched waves discussed in Fig. 2 of the main article. As before, we consider anyons on a 1D lattice with exchange phase θ_j that map onto bosons with occupation- and site- dependent Peierls phase, described by the Hamiltonian

$$\hat{H} = -J \sum_j \left(\hat{b}_j^\dagger \hat{b}_{j+1} e^{\mathrm{i}\theta_j \hat{n}_j} + \mathrm{H.c.} \right) + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) \,, \tag{S1}$$

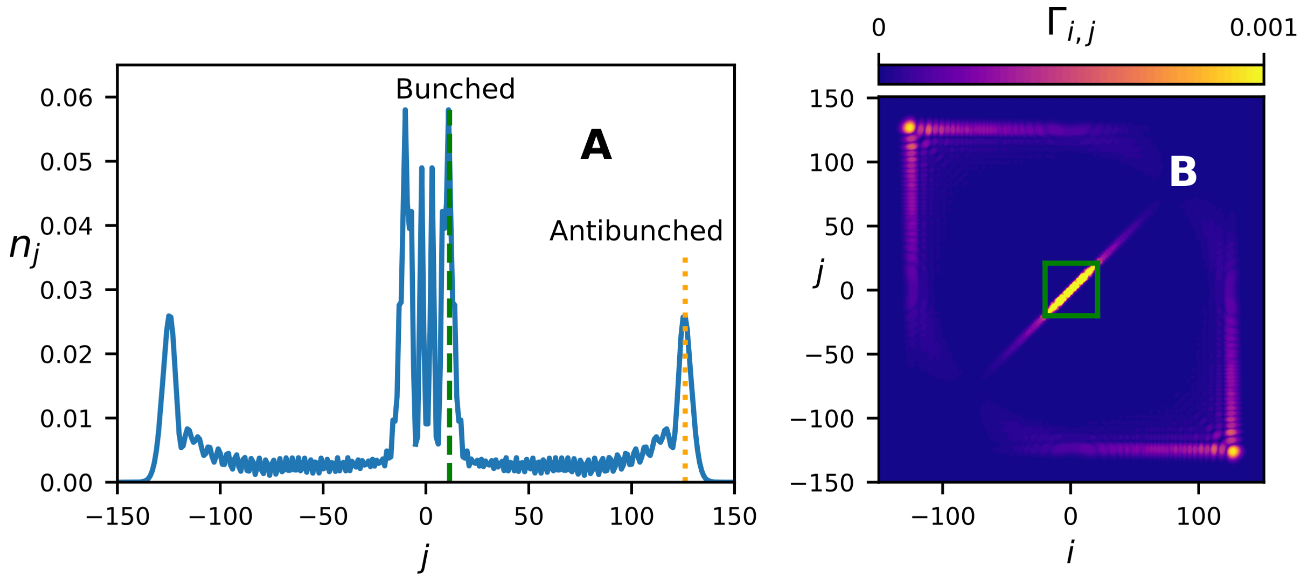

FIG. S1. Superposition of fast antibunched and slow bunched propagation with exchange phase $\theta_j = \pi$ at $Jt = 65$ after two anyons with $U = 0$ are released from sites $j = 0, 1$. (A) Density profile showing a slow bunched mode, with sharp leading edges (dashed line), and a fast antibunched mode, peaked at $j = \pm 2Jt$ (dotted line). (B) Density-density correlations showing a clear separation between the antibunched wavefront and strongly bound bunched waves within the green square.

where J is the tunneling, U is the interaction, $\hat{b}_j^\dagger$ and $\hat{b}_j$ are boson creation and annihilation operators, and $\hat{n}_j := \hat{b}_j^\dagger \hat{b}_j$. We consider anyons with $U = 0$ in the uniform case $\theta_j = \phi$, where nonzero ϕ leads to a separation of timescales in the evolution: As shown in Fig. S1, when two particles are released from neighboring sites, the dynamics split into a fast antibunched wave, where the two anyons travel in opposite directions at speed $2J$, and a slow bunched wave, where the anyons are strongly bound and propagate more slowly.

We use the leading edge of the bunched wave in the density profile (Fig. S1A) to characterize its speed as a function of the statistical phase ϕ. As shown in Fig. S2A, this speed falls off linearly from $v_{\text{slow}} = 2J$ at $\phi = 0$ to $v_{\text{slow}} \approx J/5$ at $\phi = \pi$. We calculate the relative weight in the bunched mode from the two-body correlations $\Gamma_{i,j} := \langle \hat{n}_i \hat{n}_j \rangle - \delta_{ij} \langle \hat{n}_j \rangle$ as $f_{\text{slow}} = \sum_{\square} \Gamma_{i,j}/2$, where $\square$ encloses the bunched waves (see Fig. S1B; note that $\sum_{i,j} \Gamma_{i,j} = 2$). Figure S2B shows that f_{slow} decreases monotonically with ϕ, saturating around 0.5. For comparison, we also plot the speed and weight using an effective repulsion instead of nonzero ϕ [S1], as discussed in the main text and elaborated in the next section.

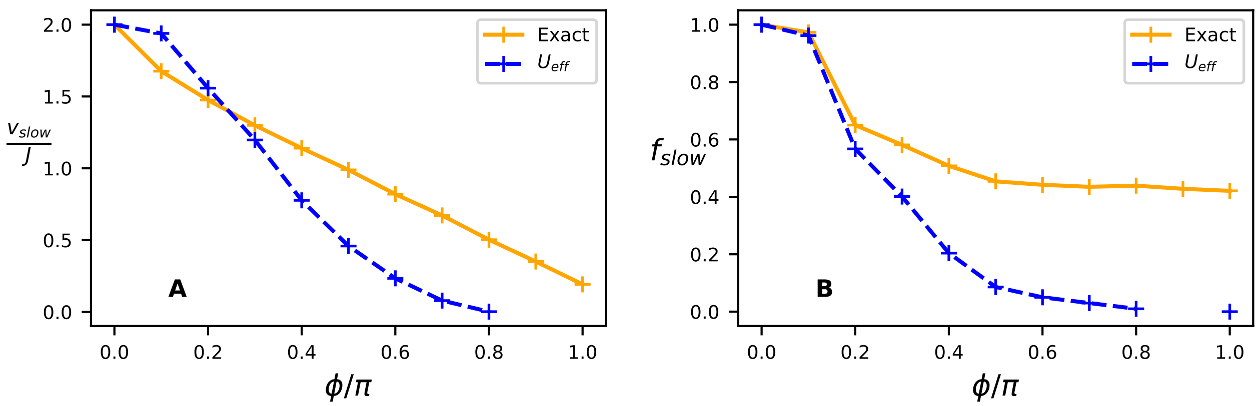

FIG. S2. (A) Propagation speed and (B) relative weight of the slow bunched waves as a function of $\theta_j = \phi$ for the exact model (solid lines) and using the effective repulsion in Eq. (S4) (dashed lines). For the latter, the bunched waves are indistinguishable from background at $\phi > 0.8\pi$. Note the strong mismatch between the two curves at large ϕ. In particular, at $\phi = \pi$, the effective repulsion describes hard-core bosons or free fermions, so double occupancy is prohibited and bunching is not supported.

SII. Effective repulsion

Here we provide quantitative comparisons between the exact dynamics and that generated by an effective interaction derived from the scattering length. As detailed in Ref. [S1], the low-energy collisions between two anyons with exchange phase ϕ is characterized by the scattering length (in units of the lattice spacing)

$$a_s = \frac{-(1+\cos\phi)}{4(1-\cos\phi) + 2U/J} , \tag{S2}$$

which can be interpreted as originating from an effective interaction strength

$$U_{\text{eff}} = \frac{4J(1-\cos\phi) + 2U}{(1+\cos\phi)} . \tag{S3}$$

Note that $U_{\text{eff}}|_{\phi\to 0} = U$. In the limit $U \to 0$, we obtain an effective repulsion as a function of the exchange phase,

$$U_{\text{eff}} = 4J \tan^2(\phi/2) , \tag{S4}$$

which can lead to slow-moving repulsively bound pairs [S2] through the effective Hamiltonian

$$H_{\text{eff}} = -J \sum_j \left(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.} \right) + \frac{U_{\text{eff}}}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) . \tag{S5}$$

However, the comparisons in Fig. S2 show this effective repulsion does not capture the full dynamics when the anyons are released from adjacent sites. In particular, for angles close to π, $U_{\rm eff}$ diverges and does not support any bunched propagation, which is a crucial feature of the model in Eq. (S1). This breakdown is not surprising since the scattering length in Eq. (S2) becomes less than the lattice spacing (in magnitude) for $\phi \gtrsim 1$, changing its interpretation [S3].

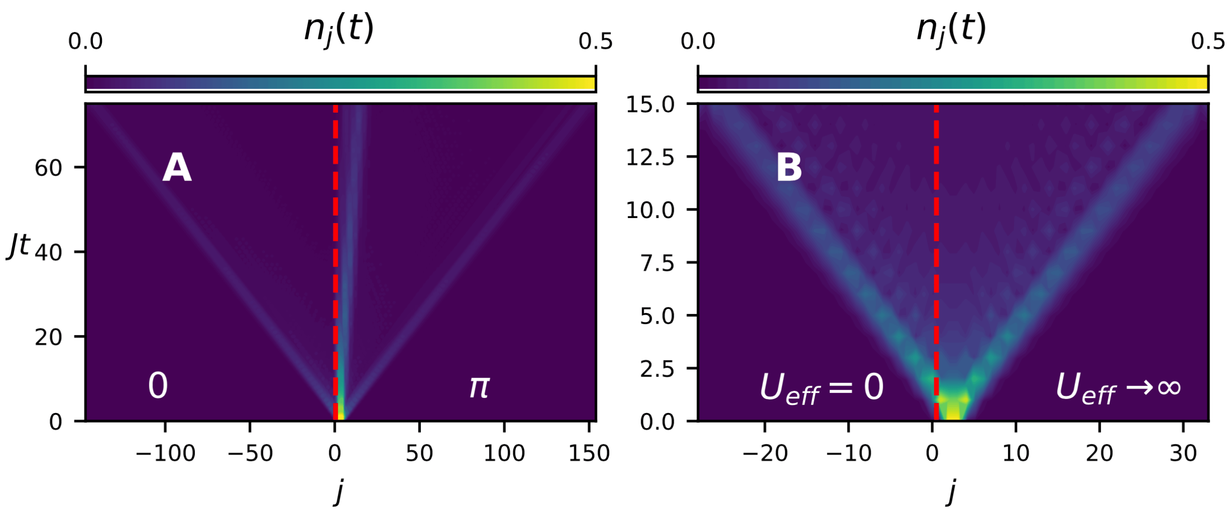

FIG. S3. Evolution of the density profile in the presence of a 0-π statistical interface ($\theta_j = \pi$ for $j > 0$), following the release of two particles from sites $j = 3, 4$ with $U = 0$. (A) Exact model [Eq. (S1)], showing a complete reflection of the slow bunched waves. (B) Using effective repulsion [Eq. (S4)], showing a fully antibunched symmetric walk that is not affected by the interface.

The above discrepancy leads to very different dynamics in the vicinity of a 0-π statistical interface when the particles are released from the pseudofermionic ($\theta_j = \pi$) side, as shown in Fig. S3. For the exact model, nearly half the initial weight goes into the slow bunched mode, which is completely reflected at the boundary, as explained in the main text. On the other hand, with the effective repulsion, the particles are fully antibunched like free fermions: they travel separately in opposite directions and are not affected by the statistical boundary.

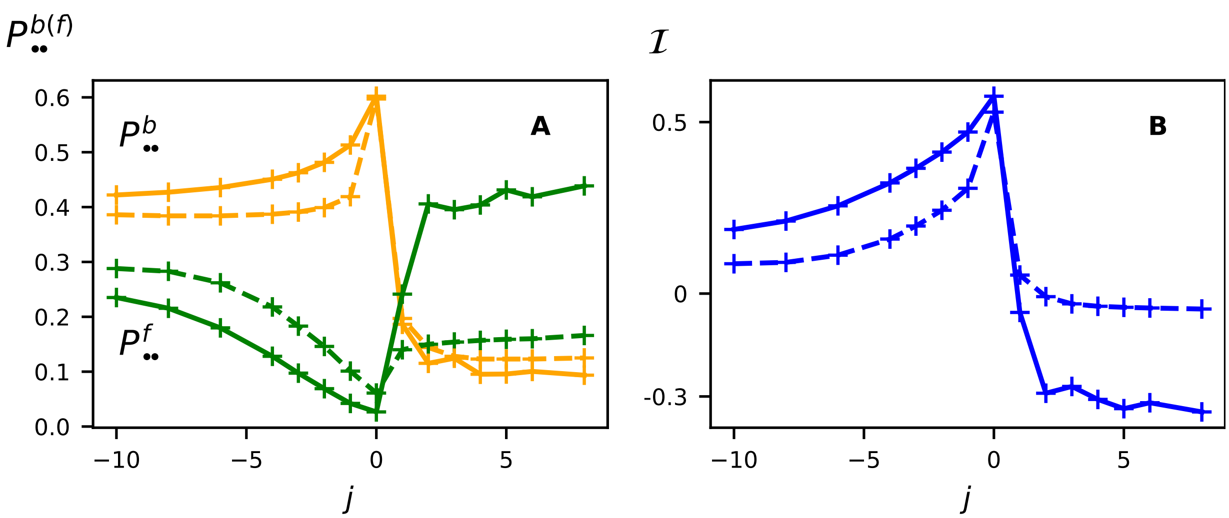

FIG. S4. Long-time asymmetries ($Jt = 100$) after the anyons are released from sites $j, j+1$ with the 0-π interface in Fig. S3 for the exact model (solid) and with corresponding effective interactions (dashed). (A) Probability of finding both particles in the left (orange) and right (green) halves. (B) Relative number imbalance between the two halves. The effective repulsion captures the qualitative features for bosonic initial states ($j \le 0$) but produces little asymmetry for pseudofermionic initial states.

In Fig. S4, we show how the long-time asymmetries in the distribution are modified in the effective-repulsion picture for different initial positions $(j, j+1)$ around a 0-π interface. As in the main text, we consider the metrics $P_{\bullet\bullet}^{b(f)}$ and $\mathcal{I}$, where $P_{\bullet\bullet}^{b(f)}$ is the probability of finding both particles in the bosonic (fermionic) side and $\mathcal{I} := (n^b - n^f)/(n^b + n^f)$ is the relative number imbalance between the two sides. Note the exact dynamics always produce stronger asymmetries. The agreement between the two is better when the particles are released from the bosonic side ($j \leq 0$) as the effective repulsion can approximately reproduce the reflection of the bunched waves. For $j > 0$, these are absent as in Fig. S3B, hence the imbalance $\mathcal{I}$ is very close to zero for the effective repulsion. Note the same-side probabilities $P_{\bullet\bullet}^{b(f)}$ are still nonzero since the antibunching is not perfect and the particles are delocalized.

SIII. Symmetric initial states with larger separation

Here we show how the effect of a statistical boundary is reduced for larger initial distance between the two particles. As before, we focus on a 0-π interface at $j = 0$. Since it alters the dynamics only by swapping the particles, we consider symmetric initial states, where two particles are released from sites $-j+1, j$, to ensure they arrive at the boundary simultaneously from the left and the right. Figure S5A shows that the interface produces an asymmetry by sending some of this incident weight as bunched waves toward the bosonic region. As the initial separation is increased, the asymmetry falls off as shown in Figs. S5B and S5C. This is because the particles have more time to delocalize before arriving at the boundary, so less weight arrives as bunched. For large separation, the particles evolve independently and the exchange phase is redundant, so $P_{\bullet\bullet}^{b(f)} \to (1/2)^2$. Note the separation is an odd number for the symmetric states, but we find a similar behavior when the particles are initially separated by an even number of sites.

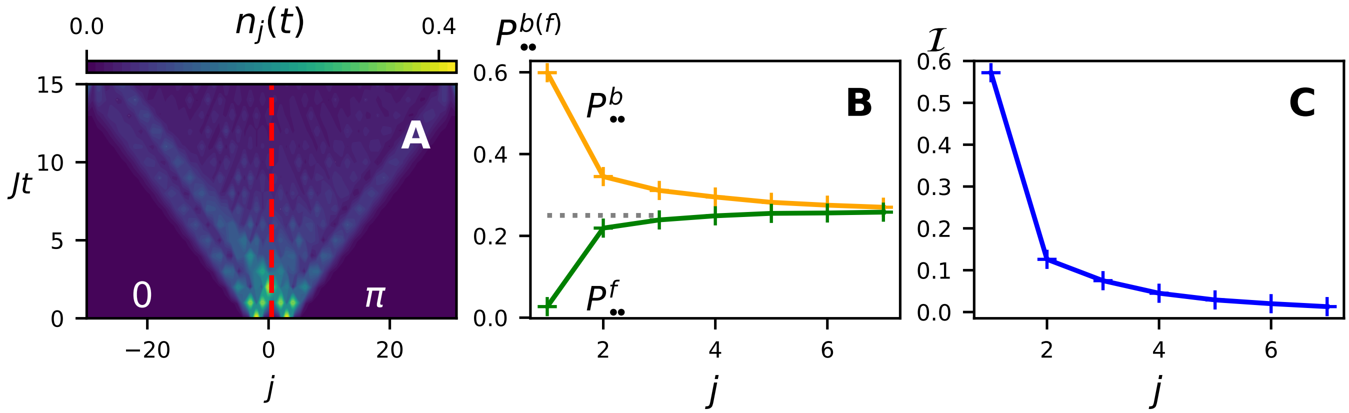

FIG. S5. Time evolution and long-time asymmetries after the anyons are released from sites $-j+1, j$ on opposite sides of a 0-π interface, with $U = 0$. (A) Density profile, showing how the interface gives rise to an asymmetry by sending bunched waves preferentially into the bosonic region. (B) Same-side probabilities $P_{\bullet\bullet}^{b(f)}$ and (C) relative number imbalance $\mathcal{I}$, showing how the asymmetry falls off with larger initial separation. For $j \to \infty$, the particles move independently, so $P_{\bullet\bullet}^{b(f)} \to 1/4$ (dotted line).

SIV. Reflections off a statistical region of finite width

Consider a junction of statistical regions α-β-γ with exchange phases (I) $\phi_\alpha = 0$, $\phi_\beta = \pi$, $\phi_\gamma \neq \pi$ or (II) $\phi_\alpha = \pi$, $\phi_\beta = 0$, $\phi_\gamma \neq 0$, and suppose the particles are released in region α. Here we show the dynamics are insensitive to the width of region β. This is expected since no bunched waves are transmitted through the α-β interface, as sketched in Fig. 4 of the main text, which makes the β-γ interface redundant. This is numerically confirmed in Fig. S6 which shows that the long-time asymmetry quickly saturates as a function of the width of region β for both (I) with $\phi_\gamma = 0$

and (II) with $\phi_\gamma = \pi$.

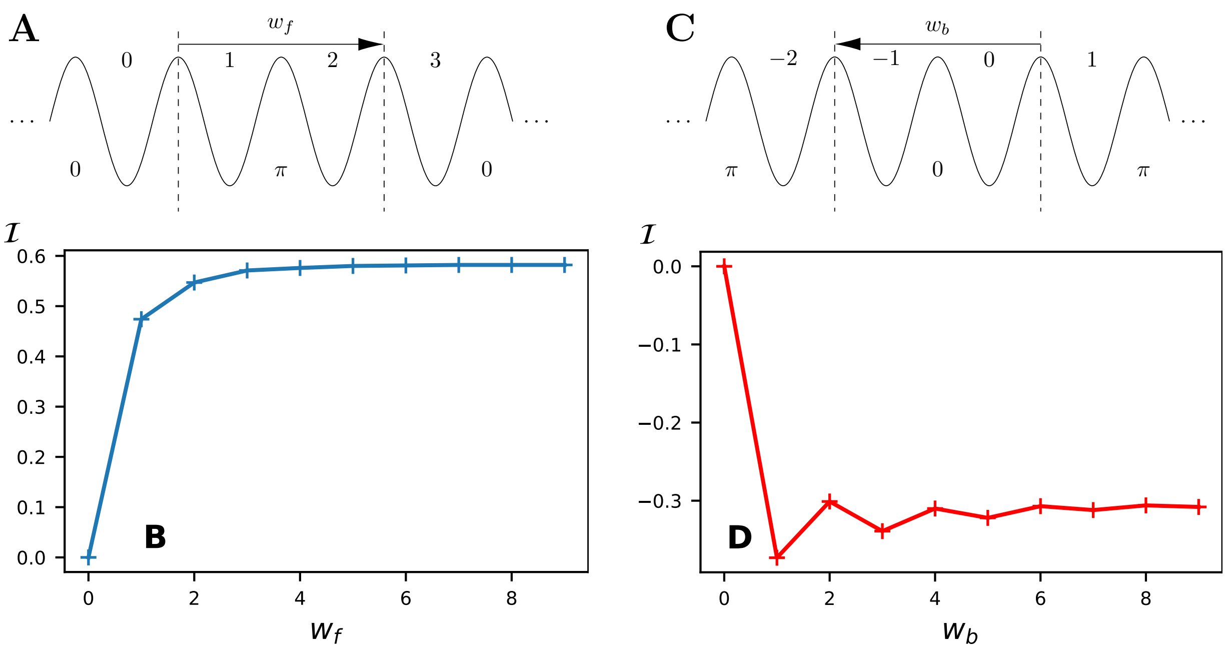

FIG. S6. Insensitivity of the dynamics of two anyons with $U = 0$ released from one side of a statistical junction to the width of the middle region. (A) 0-π-0 interface; anyons released from $j = 0, 1$, and (C) π-0-π interface; anyons released from $j = 2, 3$. (B) and (D) show the corresponding number imbalance at long times ($Jt = 100$) between the regions $j \leq 0$ and $j > 0$. The imbalance saturates to a nonzero value as soon as the middle region spans two or more sites.

[S1] S. Greschner and L. Santos, Anyon Hubbard model in one-dimensional optical lattices, Phys. Rev. Lett. **115**, 053002 (2015).

[S2] P. M. Preiss, R. Ma, M. E. Tai, A. Lukin, M. Rispoli, P. Zupancic, Y. Lahini, R. Islam, and M. Greiner, Strongly correlated quantum walks in optical lattices, Science **347**, 1229 (2015).

[S3] A. K. Kolezhuk, F. Heidrich-Meisner, S. Greschner, and T. Vekua, Frustrated spin chains in strong magnetic field: Dilute two-component Bose gas regime, Phys. Rev. B **85**, 064420 (2012).