Pulsed Generation of Symmetry-Protected Long-Range Entanglement

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SD, Stefan Kuhr, & Nigel Cooper, arXiv 2201.10564

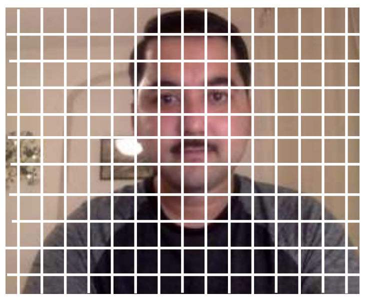
EPSRC Engineering and Physical Sciences Research Council

DesOEQ

SIMONS FOUNDATION

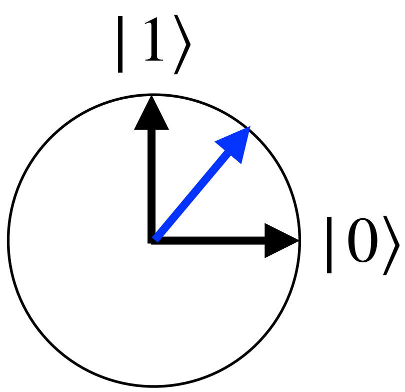
Qubits and Entanglement

Classical bit — unit of information: 0 or 1



Data stream $\ldots 0010110101\ldots$

Quantum bit ("qubit") — can be in a superposition $\alpha|0\rangle + \beta|1\rangle$



Measurement gives:

$|0\rangle$: probability $|\alpha|^2$

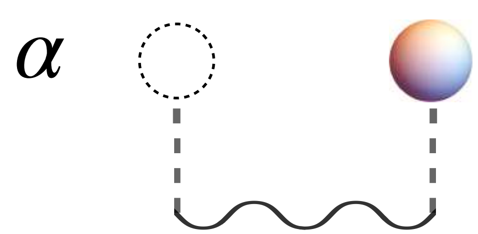
$|1\rangle$: probability $|\beta|^2$

Spin: $\alpha\,|\downarrow\rangle + \beta\,|\uparrow\rangle$

Lattice site: $\alpha\,|◌\rangle + \beta\,|●\rangle$

Entangled pair of qubits $\alpha|01\rangle + \beta|10\rangle$

$|\alpha| = |\beta|$: "Bell pair"



Qubits and Entanglement

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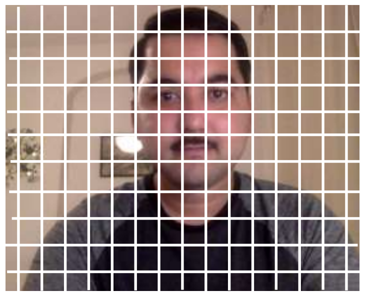

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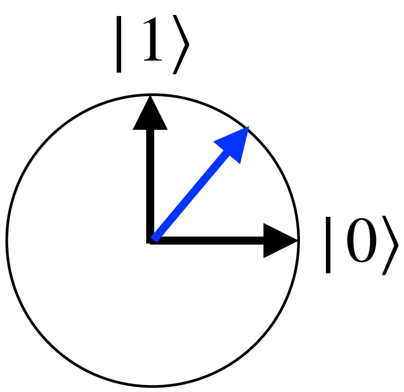

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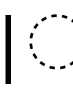
Lattice site: α |  ⟩ + β |  ⟩

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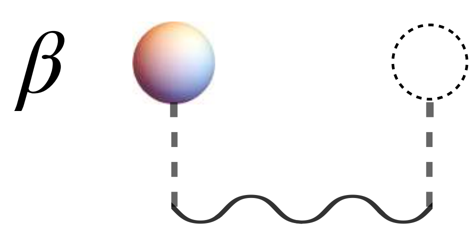

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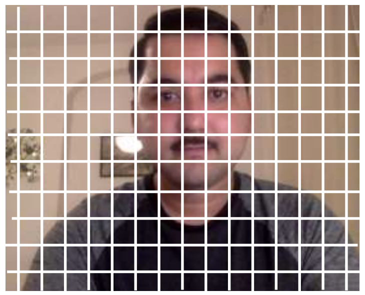

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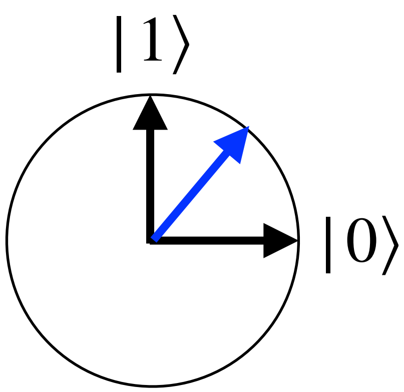

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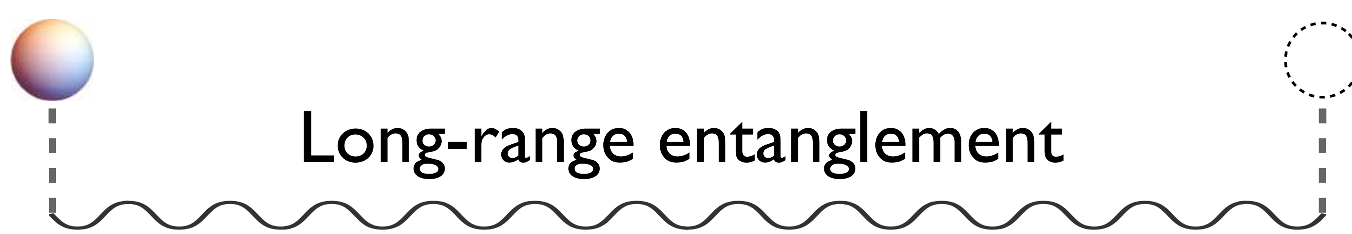
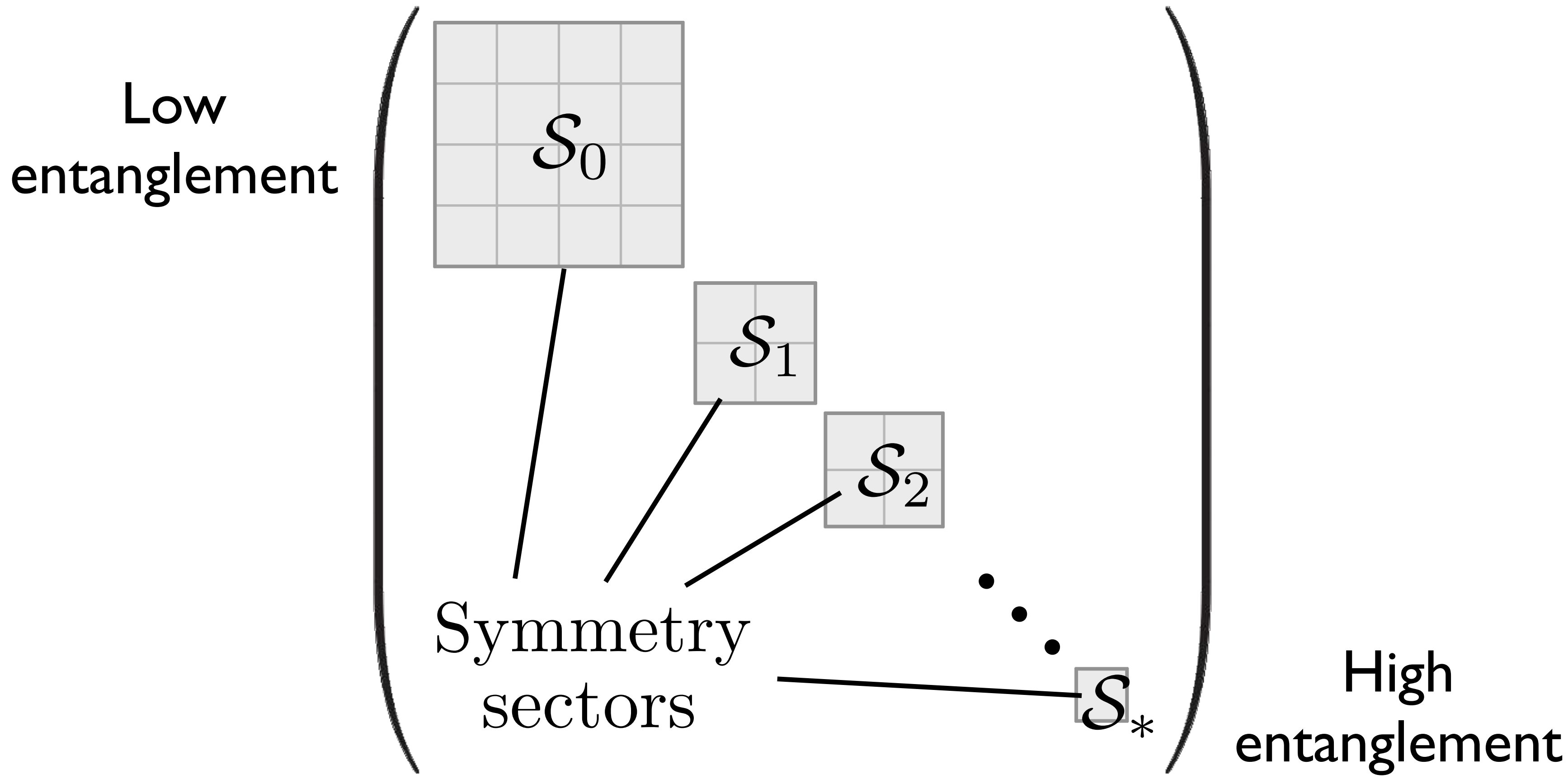


Hard to prepare in a generic many-body environment

Principle

Harness symmetry to generate long-range entanglement



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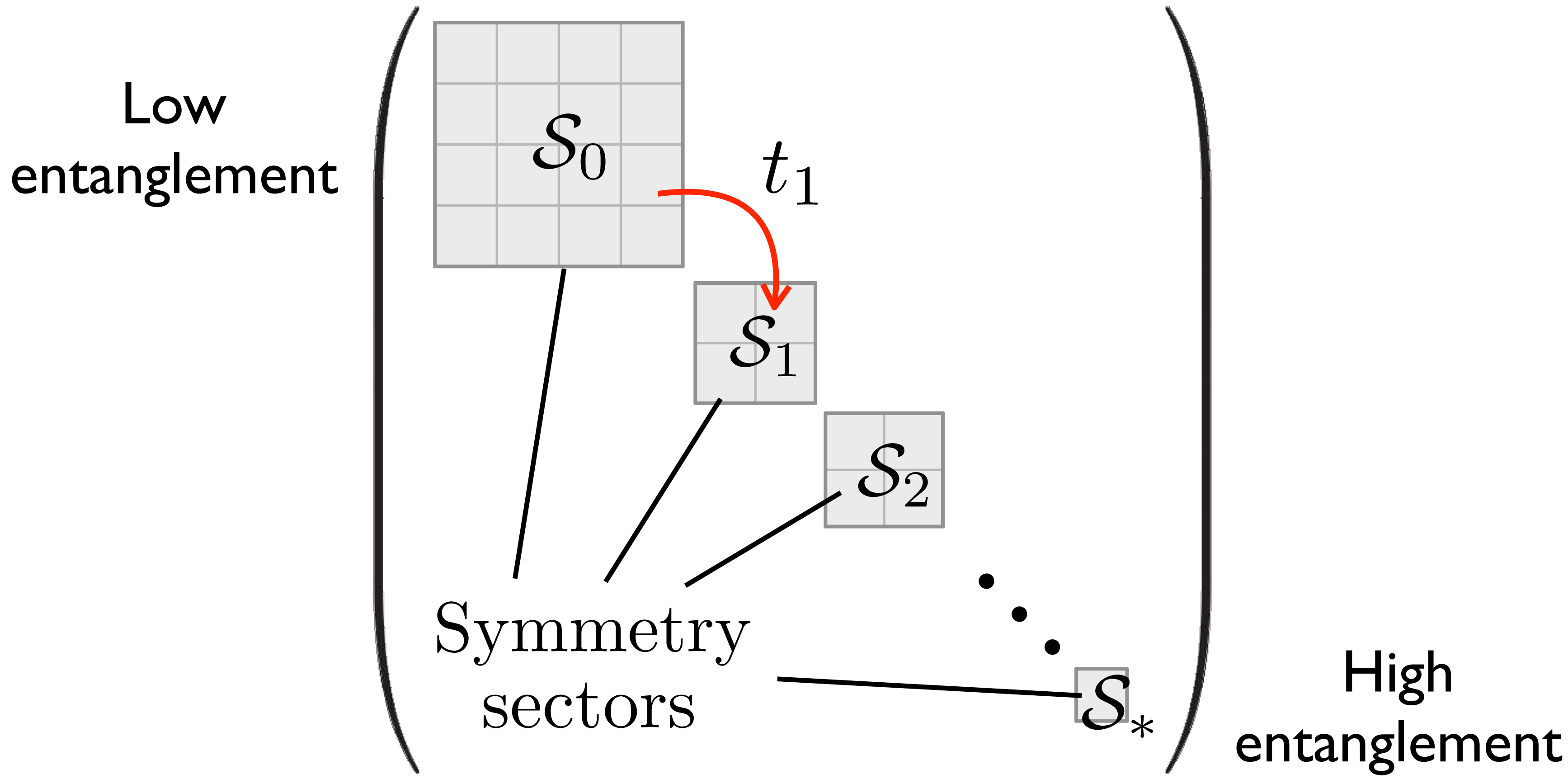

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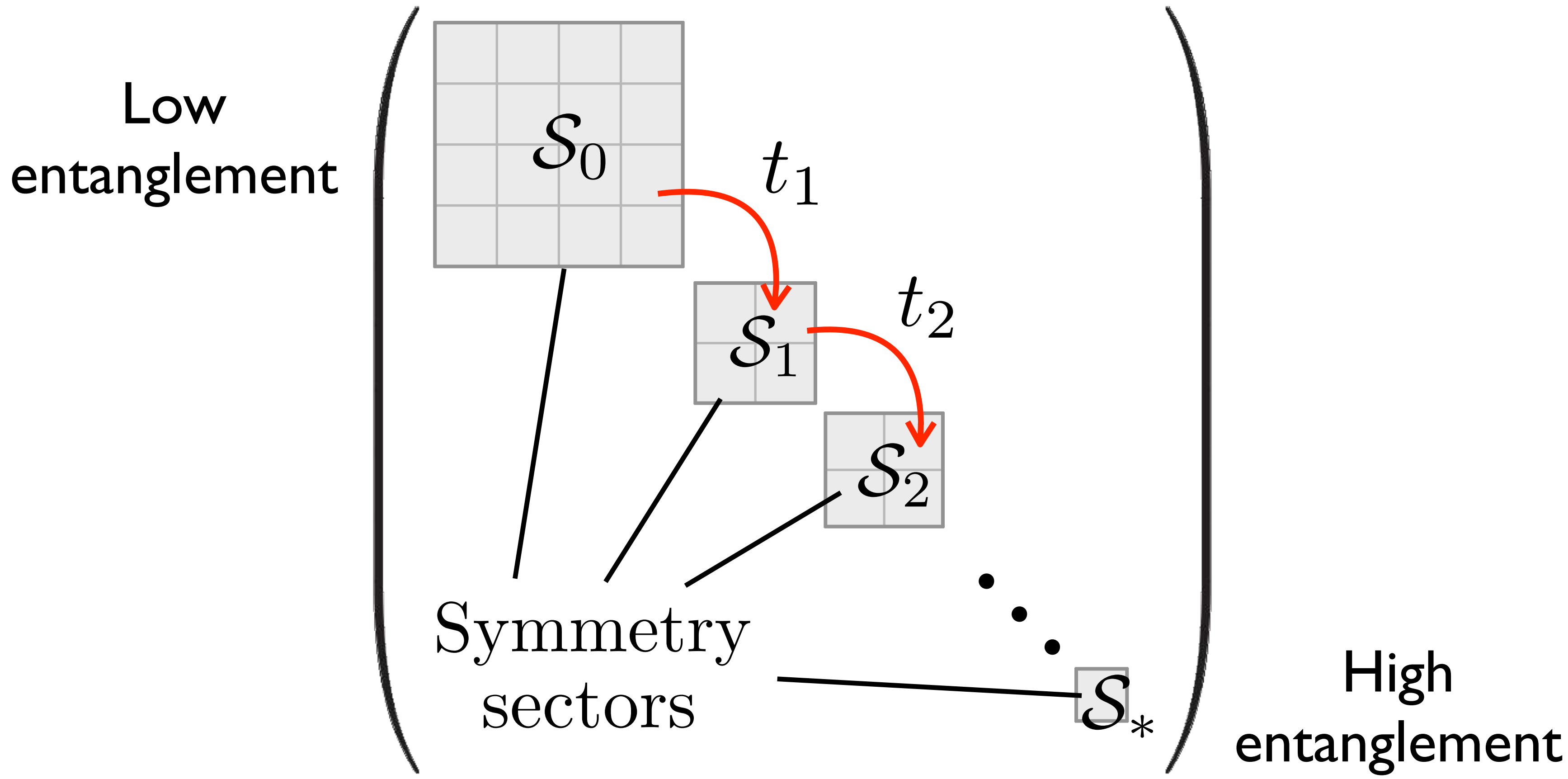

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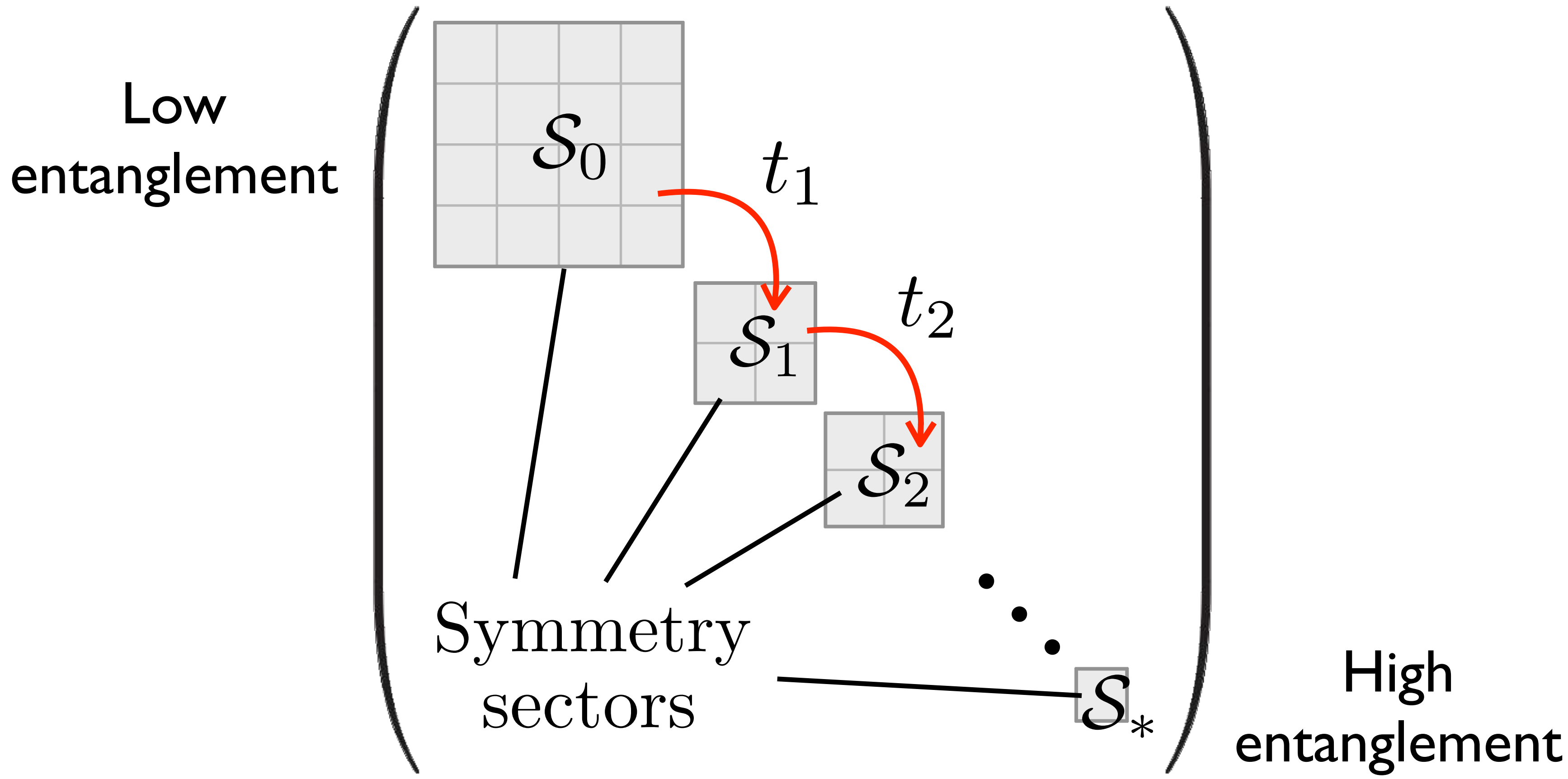

⇒ On-demand generation of entanglement and quantum correlations

Qubit array

Spin-1/2 XX chain $\hat{H} = -J\sum_i \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y$

Cold atoms/ions,
superconducting circuits…

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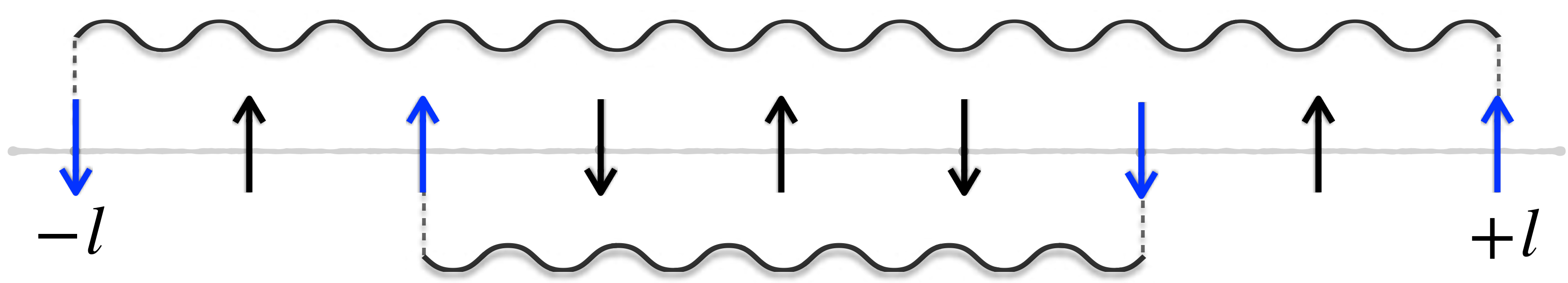
Symmetry that counts Bell pairs between sites $\{i, -i\}$

PRL **125**, 240404 (2020)

$[\hat{H}, \hat{C}] = 0$ $\hat{C} = \hat{\sigma}_0^z/2 + \sum_{i\neq 0} \hat{f}_i^\dagger \hat{f}_{-i}$

$\hat{f}_i := \hat{\sigma}_i^- \prod_{j<i} \hat{\sigma}_j^z$



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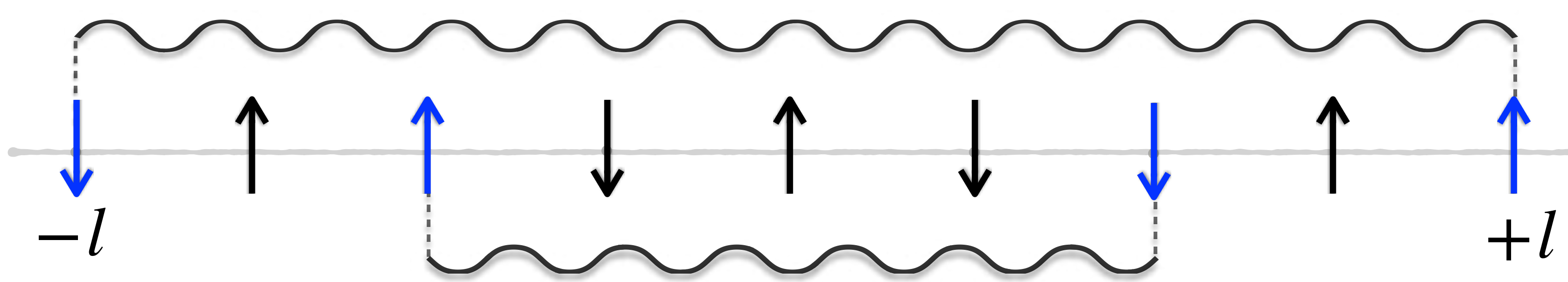
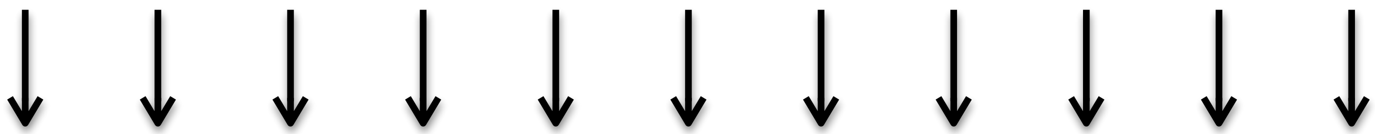


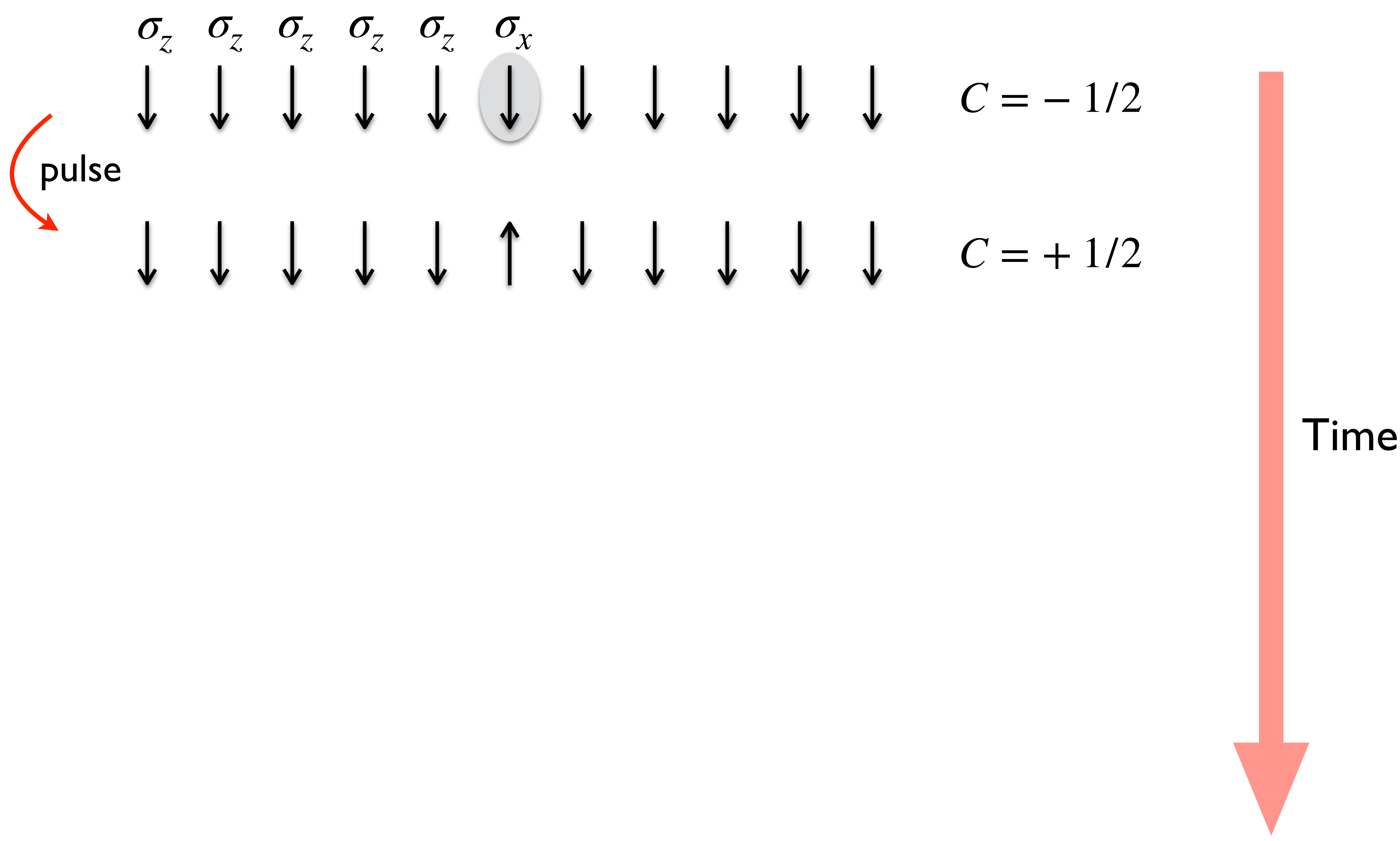
Strategy: Produce l Bell pairs by pumping C

Procedure



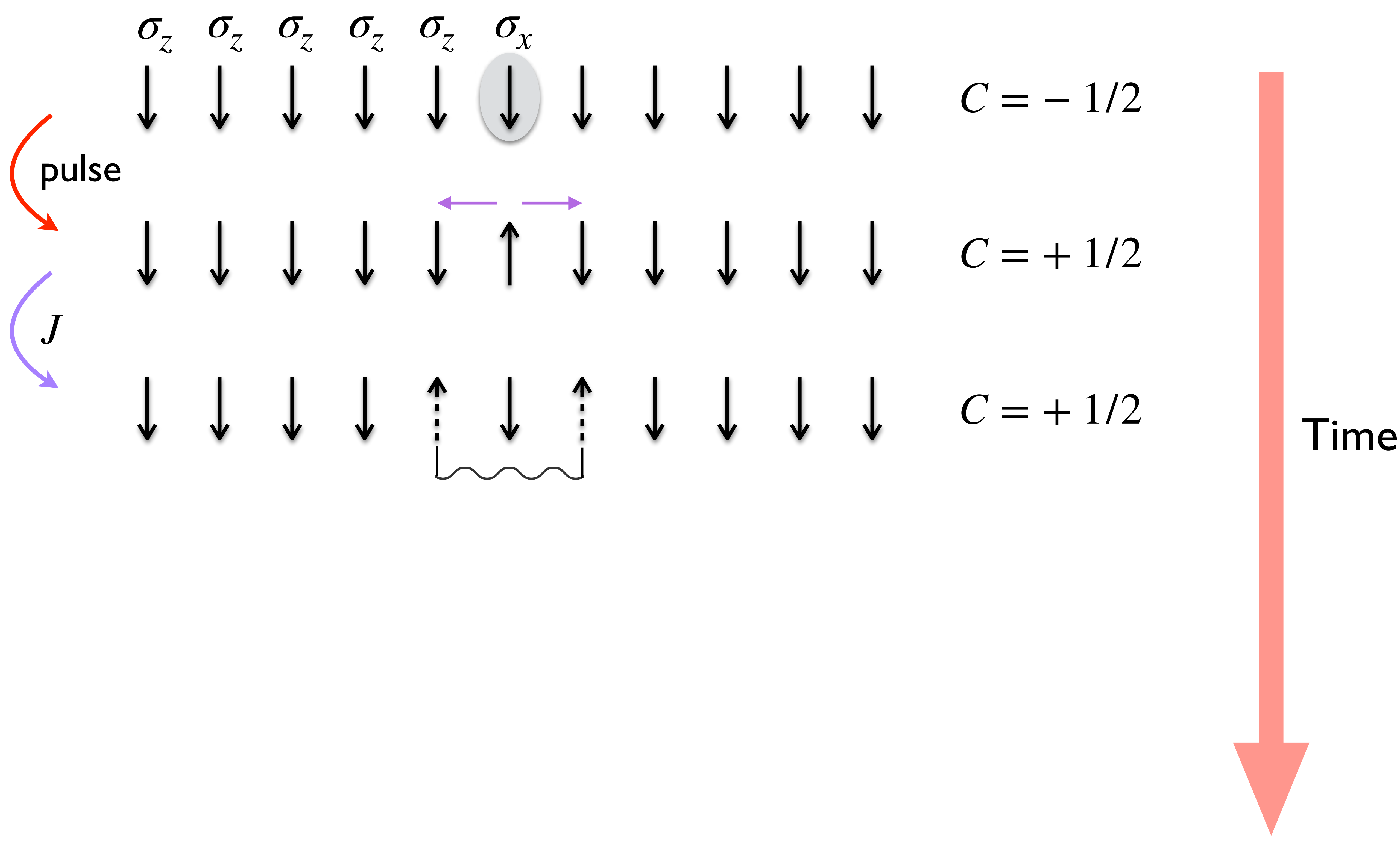
$C = -1/2$

Procedure



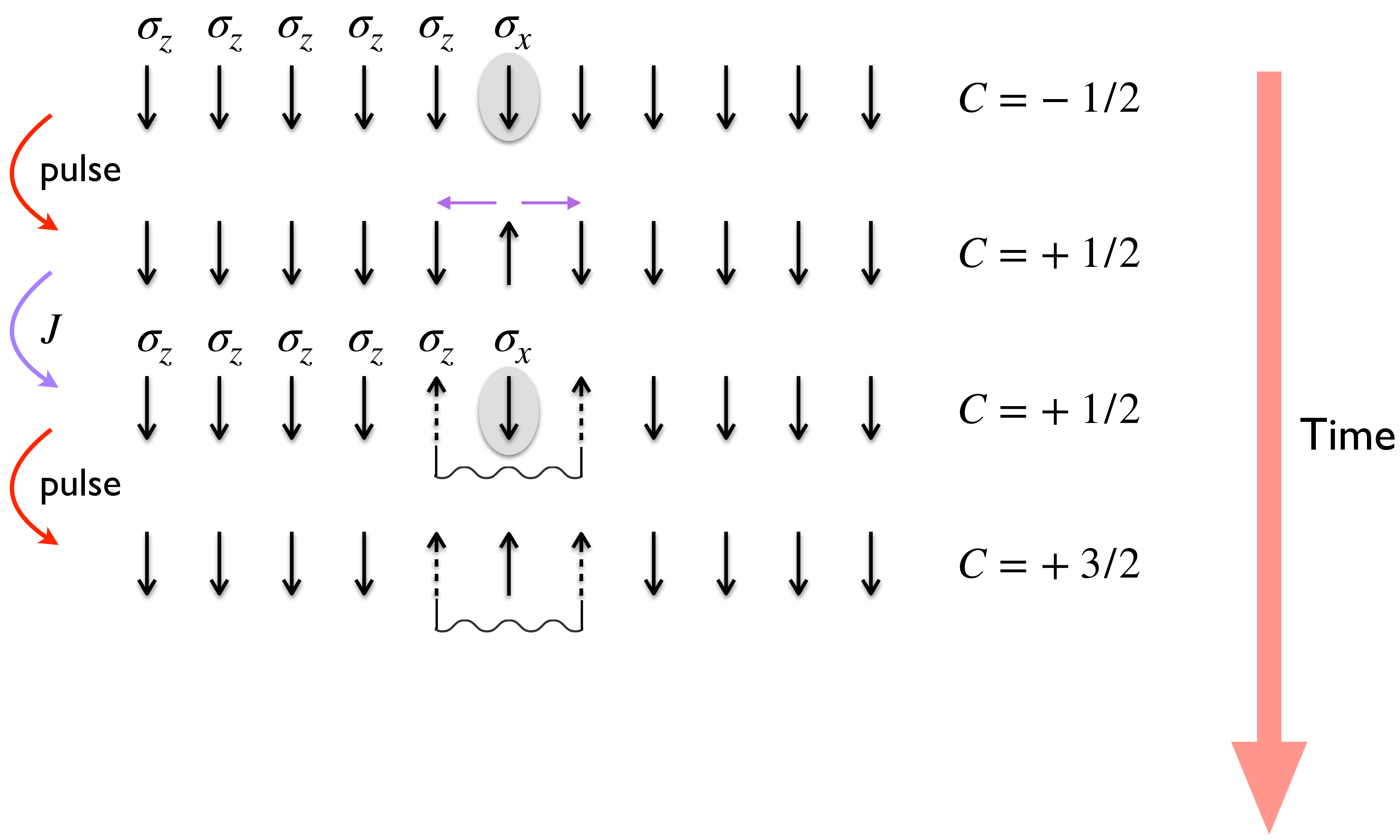
σ_z σ_z σ_z σ_z σ_z σ_x
$C = -1/2$
pulse
$C = +1/2$
Time

Procedure



σ_z σ_z σ_z σ_z σ_z σ_x
$C = -1/2$
pulse
$C = +1/2$
J
$C = +1/2$
Time

Procedure



σ_z σ_z σ_z σ_z σ_z σ_x
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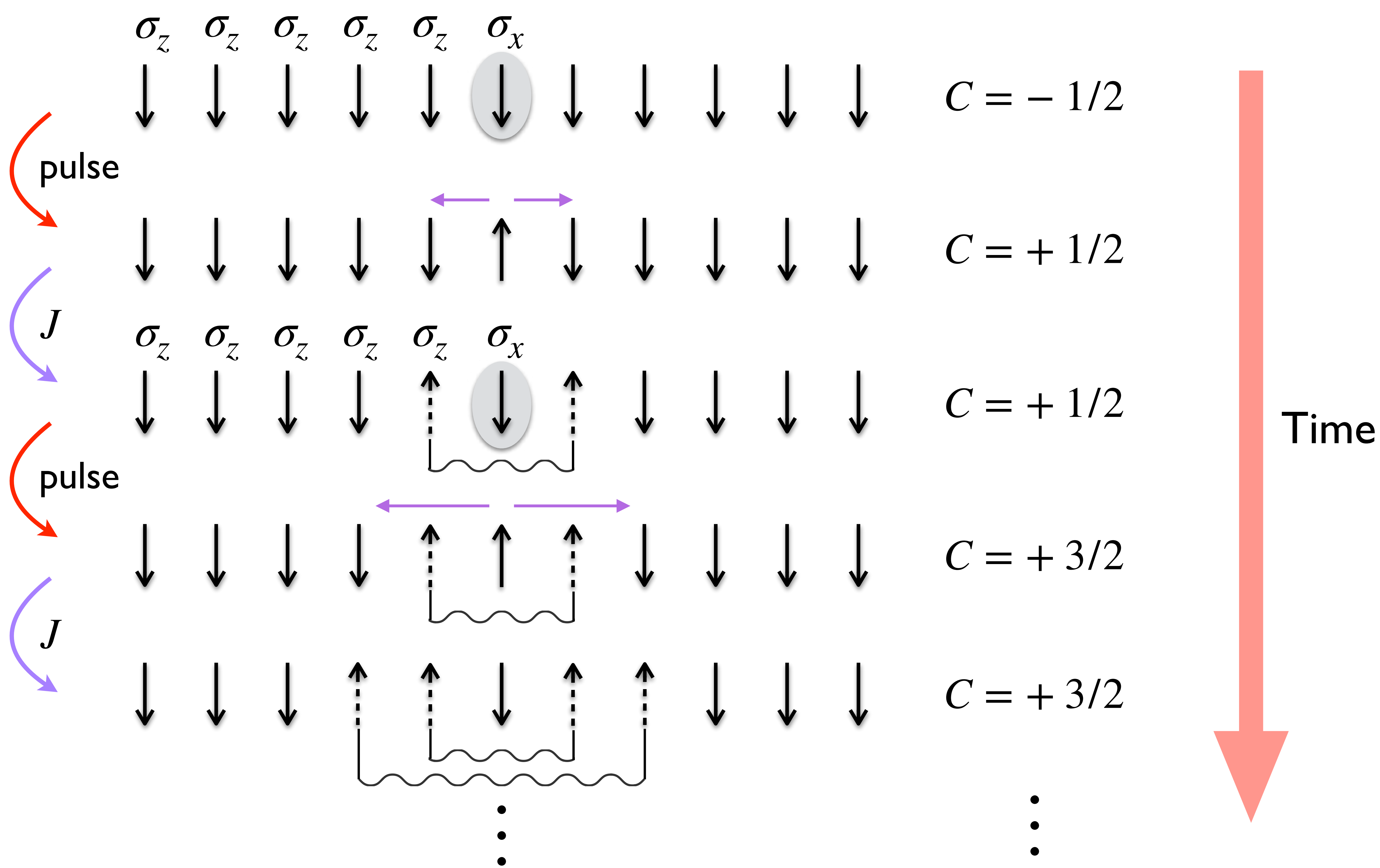
pulse
$C = +1/2$
J
σ_z σ_z σ_z σ_z σ_z σ_x
$C = +1/2$
pulse
$C = +3/2$
Time

Procedure



σ_z σ_z σ_z σ_z σ_z σ_x
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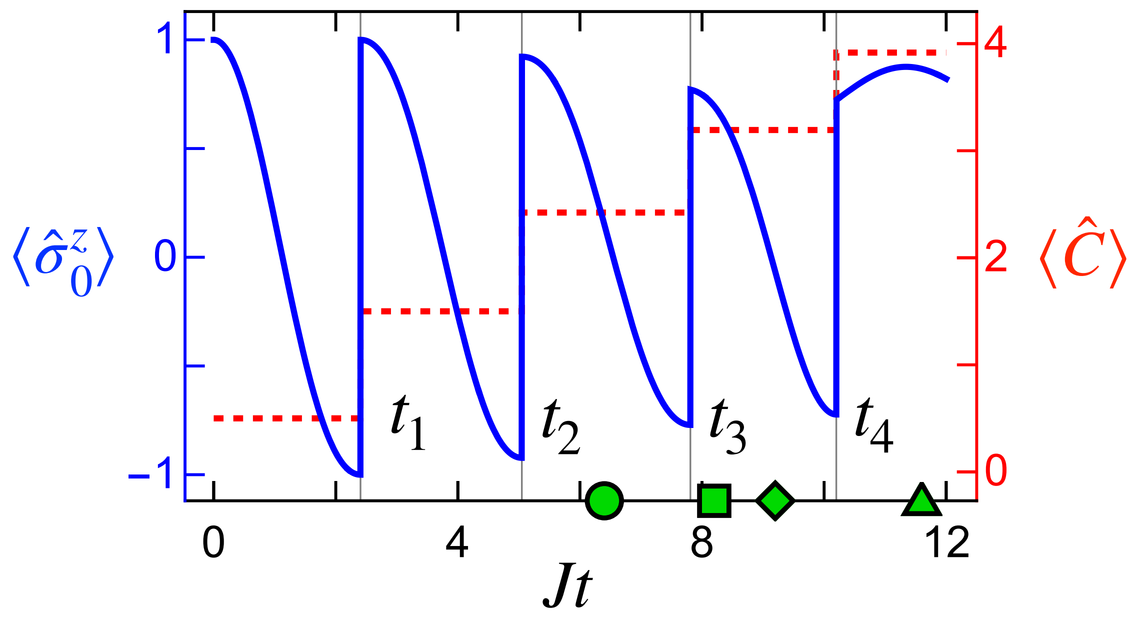
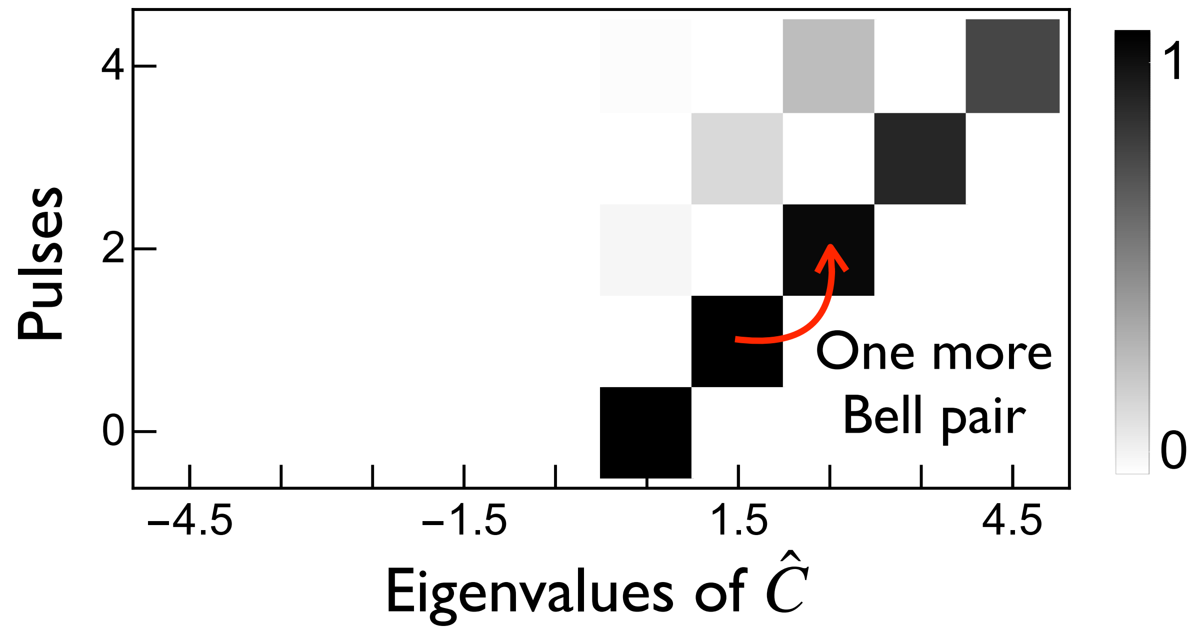
Time

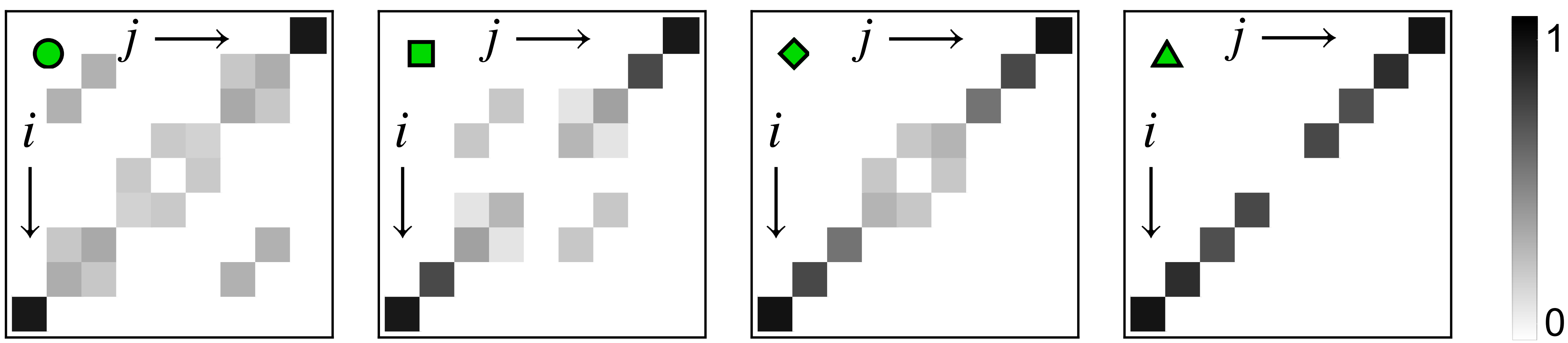
Result — Exact diagonalisation

Pulse whenever centre site is mostly ↓, up to l times





Concurrences $\mathscr{C}_{i,j}$ show Bell pairs are successively stacked inward



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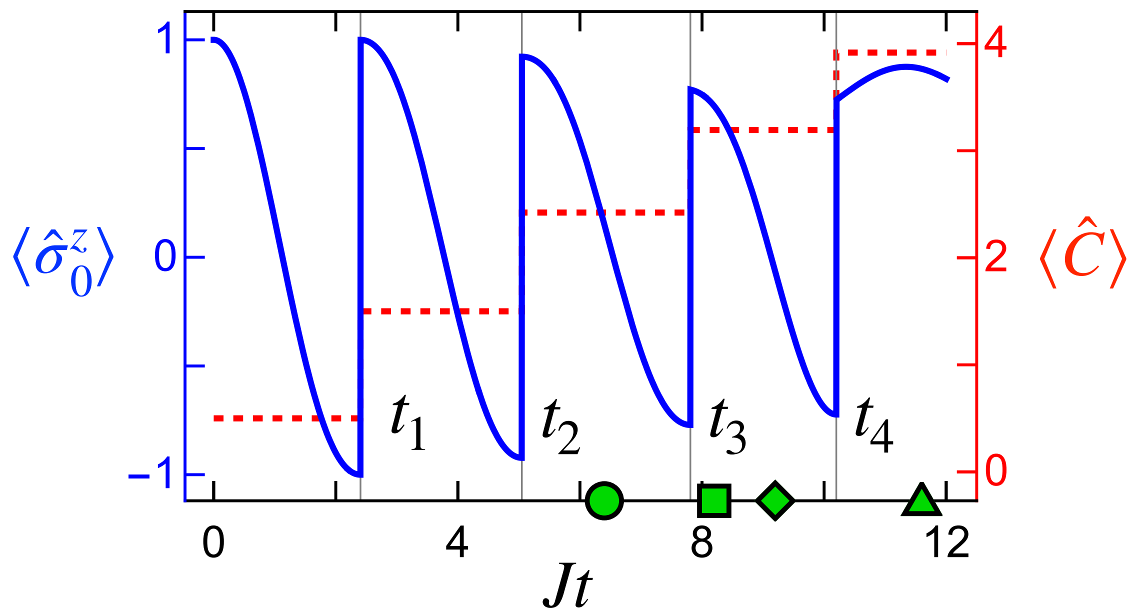

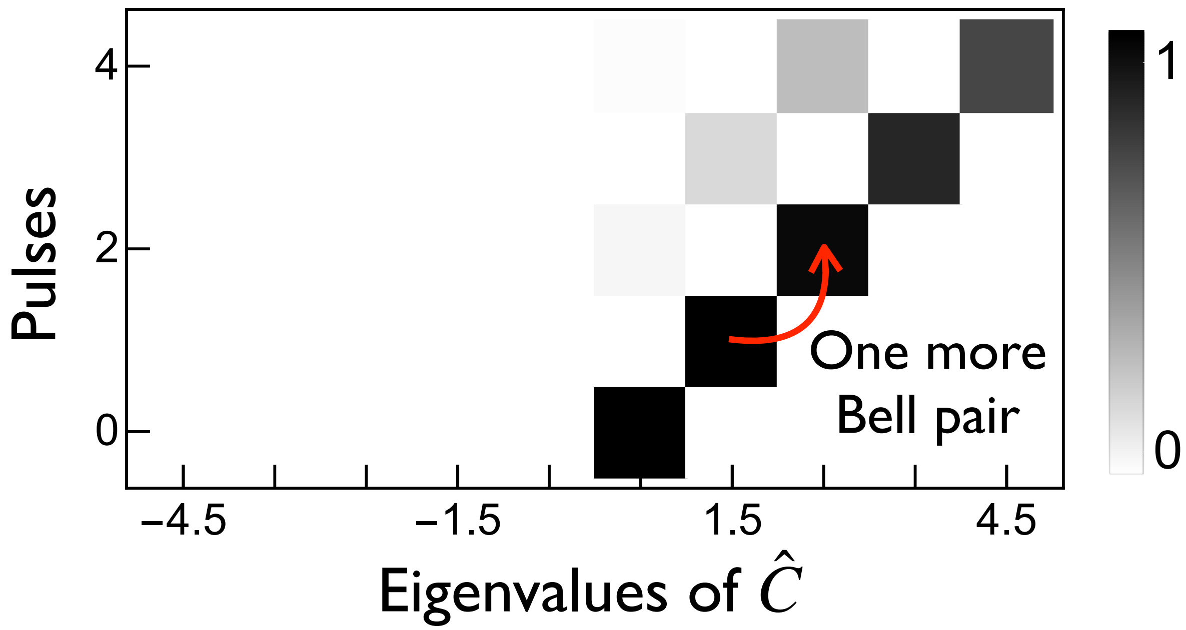

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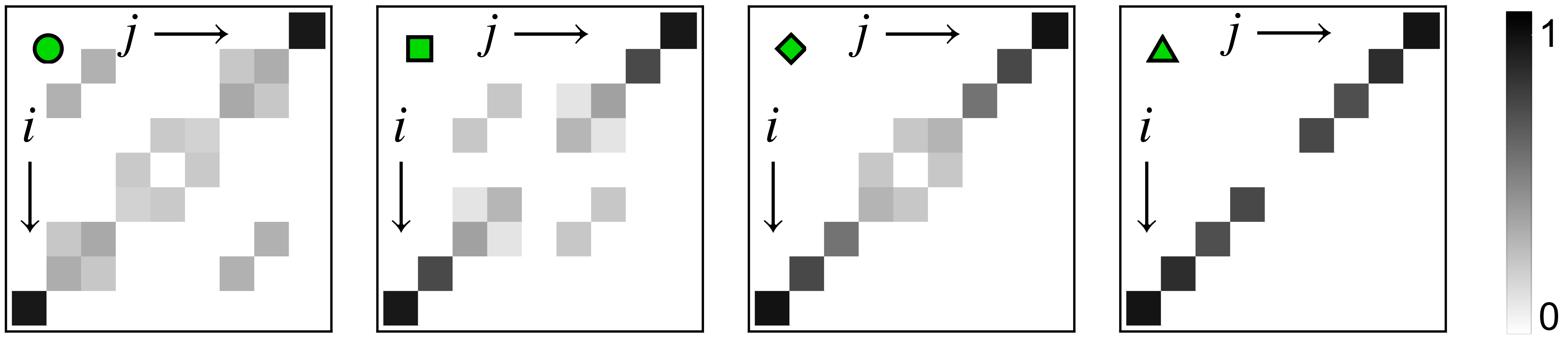

For large l — number of Bell pairs grows linearly, $\langle\hat{C}\rangle \sim 0.6l + 1$

— preparation time grows linearly, $\Delta t \sim 2l/J$

Fermi-Hubbard model — η pairs

Hamiltonian $\hat{H} = \sum_{s=\uparrow,\downarrow} \sum_{i=-l}^{l-1} \left(-J\, \hat{c}_{i,s}^{\dagger} \hat{c}_{i+1,s} + \textbf{h.c.} \right) + U \sum_{i=-l}^{l} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$

SU(2) symmetry: $\hat{\eta}^{-} = \sum_i (-1)^i \hat{c}_{i,\uparrow} \hat{c}_{i,\downarrow}, \quad \hat{\eta}^{+} = \hat{\eta}^{-\dagger}, \quad \hat{\eta}^{z} = \sum_i (\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} - 1)/2$

- $\hat{\eta}^{+} | \mathrm{vac} \rangle$ — doublon w/ quasimomentum π (η pair) — long-range coherence
- At half filling, $\hat{\eta}^2 = \hat{\eta}^{+} \hat{\eta}^{-}$ counts η pairs — eigenvals $N_\eta (N_\eta + 1), \; N_\eta = 0, 1, \ldots, l$
- Neighbouring sectors coupled by current operator $\hat{\mathcal{J}}$ (imaginary tunnelling)

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Starting from ground state ($N_\eta = 0$), apply $\hat{\mathcal{J}}$ when it will increase $\langle \hat{\eta}^2 \rangle$ the most